Course Objectives

This course aims to provide students with a basic familiarity with the commonest tools used in analysing ordered data, such as time series: this type of data is ubiquitous in geophysics and oceanography. The aim is to provide the level of understanding required to work with such data, and also to read the literature devoted to such tools.

The course focuses particularly on the use of Fourier methods to analyze and understand data and the processes which produce them; experience shows both that these methods can be very effective, and that they are often not obvious when first encountered.

Because there are a wide range of methods in use for Fourier analysis, an important part of the course is presenting those that are most flexible and least likely to lead to error.

Prerequisites

The course is mathematical, but much of it requires no more than the usual undergraduate level in science and engineering, namely ease in dealing with complex variables and calculus in one variable. The second half requires familiarity with statistics at an intermediate level, including some basic concepts of estimation theory. For SIO graduate students the usual way to acquire this is by taking SIO 210A.

Reading, Class Activities, and Homework

The course is based on, and will be taught from, class notes provided by the instructor, sent to class members by email. There is a class Web page but it is not currently being used. The notes include a bibliography of suggested other books on the course topics, many going into more depth than is possible for a one-quarter course.

There will be a weekly homework assignment, handed out at least a week before the due date. Students should collaborate on homework as little as possible, but as much as necessary. If possible, homework should be printed from PDF files created from LaTeX (not emailed). When computer code is needed, it should be included, with appropriate commenting.

In-class time will be spent, as much as possible, in discussions, which all students should be ready to contribute to by having done the appropriate reading from the course notes in advance. On the day that homework is due, students should expect to be asked to present their work to the class. The following class will be devoted to the topics covered by the next homework assignment, and the one after that to the same topics, but with questions based on initial examination of that homework.

The grade will be based on the performance on the homework.
Course Topics

The course is divided into two parts. The first is devoted to introducing Fourier transforms for both continuous functions and discrete sequences, and how to apply the resulting concepts to the methods of digital signal processing, which can be used for any ordered sequence. The second half deals with spectrum analysis, the problem of estimating the frequency content of an ordered series whose values are assumed to be all, or in part, random.

In more detail, the sequence of topics is as follows (italics for topics covered in the notes but only discussed if time permits):

• Orthogonality for functions, specifically for sines and cosines, leading to the Fourier series for periodic functions, and Gibbs’ phenomenon. Linear time-invariant systems, convolution, and how sinusoids are best suited to analyzing such systems.

• The Fourier transform, and theorems relating various properties of functions and their transforms. The convolution theorem. Generalized functions and their importance in Fourier theory.

• The Fourier transform in more than one dimension for Cartesian coordinates, and the Hankel transform.

• Discrete-time sequences and operations, including convolution. The Fourier transform for infinite sequences and the discrete Fourier transform for finite ones. Fourier theorems for the discrete transform, and circular convolution.

• Data collection and the sampling of continuous functions. The Nyquist theorem, aliasing, decimation, and quantization error.

• Digital filters, especially Finite Impulse Response filters for frequency selection, and methods of designing these. Similar designs for differentiators, digital Hilbert transforms, and minimum-phase filters.

• Simulating lumped-parameter systems with digital filters. Using the Laplace transform and poles and zeros to describe and understand a system. The z transform for digital sequences and filters, and the design of recursive digital filters.

• Sequences of random variables, in particular stationary processes, and how to describe these by the autocovariance function (or sequence).

• The power spectral density of a stationary random function, its relationship to the autocovariance, other ways to define it, and the appropriate dimensions and units for it. Properties of the spectrum for sampled data and its definition for stationary random sequences.

• The simplest estimator for the power spectrum, the periodogram: its statistics and unsuitability as an estimator. Simple ways to improve it with tapers and averaging.

• Multitaper estimations of the power spectrum. Two families of tapers: first, the minimum-leakage tapers (Slepian functions) and their use; second, local bias minimization and adaptive multitaper estimates.

• Prewhitening of series using prediction-error filters to reduce bias. Designing such filters.

• Statistical descriptions for pairs of random data, pairs of random sinusoids, and pairs of stationary signals. The cross-spectrum and methods of estimating it, and some sources of bias.

• Stationary processes in the plane and the power spectrum for these. Effect of using a profile.