

## SIO229 – Gravity and Geomagnetism

Winter 2014

Mon/Wed/Fri 9:30~10:20 am

Munk Conference Room, IGPP

### Instructors:

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This web page is a repository for class notes, assignments, and other items of possible interest for SIO229, a 4-unit graduate class on gravity and geomagnetism. Required course reading will be in the form of notes (authored by Bob Parker and Cathy Constable) posted below in PDF format, with assigned sections for each lecture.

The class is an introduction to potential theory in the context of gravity and geomagnetism at a level suitable for the advanced nonspecialist in geophysics. The subtext is to get you familiar with spherical harmonics, which are ubiquitous functions in global geophysics. After a brief warm-up we are going to introduce spherical harmonics, derive some of their properties and apply them to various problems in potential fields, with emphasis on the geoid. Along the way we will meet some old topics, like the precession of the equinoxes and classical problems of potential theory, as well as some newer ones, like the satellite-derived geoid and gravity models of other bodies in the solar system.

Theoretical topics include spherical harmonics, Laplace's equation, boundary-value problems on a sphere, and Fourier methods. Applied topics include the global geoid, gravity anomalies, geomagnetic field modeling and sources, and paleomagnetic observations.

Prerequisite: graduate standing or consent of instructors. Grading for this class will be based on homework assignments (60%), short presentations of topics from the text or homework (15%) and a final exam (25%).

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### Course Materials (Gravity):

[Gravity Notes #1 \(pages 1-16\)](#)

[Gravity Notes #2 \(pages 17-26\)](#)

[Gravity Notes #3 \(pages 27-36\)](#)

[Gravity Notes #4 \(pages 37-50\)](#)

[Gravity Notes #5 \(pages 51-65\)](#)

[Appendix A – Some Vector Identities](#)

[Appendix B – Orthogonality of Spherical Harmonics](#)

[Appendix D – Random Density Fluctuations](#)

[Appendix Q – Numerical Calculus](#)

[Appendix B – Orthogonality of Spherical Harmonics](#)

### Gravity Lecture Contents:

#### Lecture 1 - 1/6/2014

Topics: Introduction to the gravity portion of the course, moments of inertia for an arbitrary body.

Reading: Gravity Notes #1, section 1

**Lecture 2 - 1/8/2014**

Topics: Principal axes, multipole expansion of the gravitational potential of Earth (MacCullagh's Formula), relationship between potential and moments of inertia.

Reading: Gravity Notes #1, sections 2~4.

**Lecture 3 - 1/10/2014**

Topics: Measurement of inertial parameters for Earth, planetary mass distributions.

Reading: Gravity Notes #1, section 4.

**Lecture 4 - 1/13/2014**

Topics: Introduction to the geoid via Clairaut's Formula.

Reading: Gravity Notes #1/#2, section 5

**Lecture 5 - 1/15/2014**

Topics: Introduction to surface spherical harmonics as solutions to Laplace's equation on unit sphere.

Reading: Gravity Notes #2, sections 6~7

Additional resources: Blakely Chapters 1, 3, 6; H&M 1-1, 1-2, 1-7 (we'll cover Stokes' theorem later), 1-9, 1-10, 1-11 (maybe 1-13, 1-14, although not as useful)

**Lecture 6 - 1/17/2014**

Topics: Hilbert spaces,  $L_2(S(1))$ , and a discussion of orthonormal bases.

Reading: Gravity Notes #2/#3, section 7; Appendix B (Orthogonality of S.H.)

**Lecture 7 - 1/22/2014**

Topics: A field guide to the spherical harmonics: behavior of zonal, sectorial and tesseral spherical harmonics from their functional forms .

Reading: Gravity Notes #2/#3, section 7

Additional resources: Blakely Chapter 6; H&M 1-9, 1-10, 1-11

**Lecture 8 - 1/24/2014**

Topics: Review of the Table of Spherical Harmonic Lore (page #29), derivation of Jean's Formula, perspectives on the concept of "wavelength" in the context of spherical harmonics.

Reading: Appendix P (Jean's Formula)

**Lecture 9 - 1/27/2014**

Topics: Solution of Laplace's equation using spherical harmonics, introduction to global geoid models.

Reading: Gravity Notes #3, section 8

**Lecture 10 - 1/29/2014**

Topics: Global geoid models, reference ellipsoids, Brun's Formula for calculating geoid height.

Reading: Gravity Notes #3/#4/#5, sections 9, 15

Additional resources: H&M 2-8, 2-11, 2-13 (note that H&M use  $T$  in place of  $\Delta U$ )

**Lecture 11 - 1/31/2014**

Topics: The gravity anomaly, boundary-value problems in potential theory, spherical harmonic solution to Brun's Formula via the gravity anomaly, Stokes' Formula.

Reading: Gravity Notes #3/#4/#5, sections 9, 13, 15

Additional resources: H&M 1-17, 2-13, 2-16 (note that H&M use  $T$  in place of  $\Delta U$ )

**Lecture 12 - 2/3/2014**

Topics: The problem with small data regions, the Remove-Restore method for incorporating long-wavelength gravity information into regional surveys, global gravity models.

Reading: Gravity Notes #5, section 17

### Lecture 13 - 2/5/2014

Topics: An application of the theory from this course to regional geoid estimation using a flat-earth approximation.

Reading: None assigned, although the less-technical material appears in DOI: 10.1029/2007JB005445.

### Lecture 14 - 2/7/2014

Topics: Topography, gravity and geoids/lunoids/aeroids/etcerooids of the terrestrial planets.

Reading: Gravity Notes #5, section 16. Treatise on Geophysics, Chapter 10.05 (Wieczorek), Gravity Notes #5, section 18

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### Supplementary Material:

#### *Physical Geodesy*

W. Heiskanen and H. Moritz

W.H. Freeman and Company, San Francisco, 1967

[On reserve in the Munk reading room.]

A thorough treatment of much of the gravity material covered in the class notes.

#### *Potential Theory in Gravity and Magnetic Applications*

Richard J. Blakely

Cambridge University Press, New York 1995

[available from ROGER (roger.ucsd.edu) in electronic format]

Another look at potential theory, specifically in the context of gravity and geomagnetism.

#### *Treatise on Geophysics, Volume 3, Geodesy*

Volume Editor: T. Herring. Editor-in-Chief: G. Schubert

Elsevier, Amsterdam, 2007

[available from ROGER (roger.ucsd.edu) in electronic format]

Technical summaries of many current topics in gravity research, written by the best in the field.