SIO 229 Gravity and Geomagnetism: Class Description and Goals

This graduate class provides an introduction to gravity and geomagnetism at a level suitable for advanced non-specialists in geophysics. Topics on gravity and magnetism are linked through the application of common methods of potential theory. The class begins with a discussion of moments of inertia in the global context, then introduces spherical harmonics as a solution to Laplace's equation, and applies them to various problems in potential theory with emphasis on the geoid. Modern satellite derived geoid models will be discussed for Earth and other bodies in the solar system.

Theoretical topics include multipole expansions, spherical harmonics, Laplace's equation, boundary-value problems on a sphere and Fourier methods for regions small enough for flat earth approximation to be valid. The geomagnetism part of the class emphasizes global processes that produce Earth's magnetic field and moves beyond potential theory to discuss its temporal variation on all time scales.

Applied topics include the global geoid, gravity anomalies, geomagnetic field modeling and sources, simple models of secular variation, and paleomagnetic observations , and how to use measurements of the magnetic field to study Earth's interior.

Grading for this class will be based primarily on homework assignments. Students may work together on assignments but all must turn in their own solutions based on their own work. Any code written to complete assignments must be submitted along with results. Short presentations of homework or other topics may be assigned, and a final exam may be given at the instructors' discretion.

Prerequisite: graduate standing or consent of instructors.

On completion of this course students should

- understand how mass distribution in planetary bodies is related to moments of inertia, how to calculate the principal axes of an inertia tensor, and how all this relates to Earth's J2, angular momentum and Chandler Wobble.

- be able to write down a general multipole expansion of Earth's gravitational potential, describe the significance of the first several terms of the expansion, and modify the general expansion to the case of an axisymmetric idealized Earth whose center of mass is at the center of the coordinate system.

- be able to describe the difference between Earth's gravitational potential (U) and geopotential (V), and define gravity in terms of either.

- be able to derive Clairaut's Formula for the shape of the geoid for a simple Earth and use the formula to answer basic questions about the shape of Earth under various scenarios (e.g. different spin rates).

- be able to write down a spherical harmonic expansion for Earth's gravitational potential and describe the form and function of the various components (e.g. the normalization term, Legendre polynomials, azimuthal sine/cosines).

- understand spherical harmonics as an orthonormal basis on the unit sphere and be able to relate them to Fourier series on the number line.

- be able to visualize the shapes of various spherical harmonics and relate them back to the degree (l) and order (m) of the expansion term.

- be able to provide a estimate of the wavelength of any term in the spherical harmonic expansion, and explain how to use Parseval's theorem to construct a power spectral density function for a particular expansion.

- list the various components of a global gravity model and some of the differences between various popular models, and be able to construct a global geoid from a global gravity model (via the appropriate scaling of expansion coefficients).

- be able to explain the physical origin of the various sources of the magnetic field: the core, lithosphere, ionosphere, magnetosphere, and the role that magnetic fields play in controlling the environment of earth and other planets.

- understand the role of classical electrodynamics in geomagnetism, including Maxwell's equations and Helmholtz's theorem which expresses the sources of a field as the divergence or curl of a vector field.

- know when the geomagnetic field can be written as the gradient of a scalar potential, and be familiar with how to construct and use basic geomagnetic field models derived from a variety of geomagnetic field measurements.

- be able to write down and explain the equations associated with the dipole part of Earth's magnetic field.

- know how to separate internal and external sources of the field.

- evaluate the magnetic field produced by lithospheric sources of magnetization.

- be able to describe the basic equations governing the dynamo and the frozen flux approximation for studying temporal evolution of the magnetic field.

Gravity Lecture Contents:

Lecture 1 - 1/4/2016
Topics: Introduction to the gravity portion of the course; moments of inertia for an arbitrary body
Reading: Gravity Notes #1, Sections 0, 1 (Slides)
Homework: Problem Set #1 (due Monday, 1/12) Solution #1

Lecture 2 - 1/6/2016
Topics: Moments of inertia for an arbitrary body; principal axes of the inertia tensor; angular moment of Earth and the Chandler Wobble.
Reading: Gravity Notes #1, Sections 1, 2 (Slides)

Lecture 3 - 1/8/2016
Topics: Multipole expansion of the gravitational potential of Earth (MacCullagh's Formula); relationship between potential and moments of inertia.
Reading: Gravity Notes #1, section 3 (Slides)

Lecture 4 - 1/11/2016
Topics: Inertial parameters for Earth via MacCullagh's Formula.
Reading: Gravity Notes #1, section 4 (Slides)
Homework: Problem Set #2 (due Friday, 1/16) Solution #2

Lecture 5 - 1/13/2016
Topics: MacCullagh's Formula in spherical coordinates; J2 and mass distribution; a quick look at the solar system – mass, spin rates, J2s.
Reading: Gravity Notes #1, section 4 (Slides)

Lecture 6 - 1/15/2016
Topics: Measuring gravity with a pendulum; Newton's method for determining the flattening of Earth; introduction to the geoid.
Reading: Gravity Notes #1/#2, section 5 (Slides)

Lecture 7 - 1/20/2016
Topics: The basic shape of the geoid via Clairaut's Formula.
Reading: Gravity Notes #1/#2, section 5 (Slides)
Homework: Problem Set #3 (due Monday, 1/26) Solution #3

Lecture 8 - 1/22/2016
Topics: Introduction to spherical harmonics via a general solution to Laplace's equation.
Reading: Gravity Notes #2, sections 6~7 (No slides for this lecture)
Additional resources: Blakely Chapters 1, 3, 6; H&M 1-1, 1-2, 1-7 (Stokes' theorem comes later), 1-9, 1-10, 1-11

Lecture 9 - 1/25/2016

Topics: Euclidean space and coordinate axes; Hilbert spaces and basis functions; spherical harmonics as an orthonormal basis for the surface of a sphere.
Reading: Gravity Notes #2/#3, section 7; Appendix B (Orthogonality of S.H.)
(No slides for this lecture)
Homework: Problem Set #4 (due Friday, 1/30)

Lecture 10 - 1/27/2016
Topics: Spherical harmonics as an orthonormal basis on a unit sphere; a field guide to the spherical harmonics.
Reading: Gravity Notes #2/#3, section 7~8; Appendix B (Orthogonality of S.H.)
(Slides)

Lecture 11 - 1/29/2016
Topics: The Table of Spherical Harmonic Lore; perspectives on the concept of "wavelength" in the context of spherical harmonics.
Reading: Gravity Notes #3, page 29; Appendix P (Jean's Formula) (Slides)

Lecture 12 - 2/1/2016
Topics: Spherical Harmonic Expansion of Earth's gravitational potential. Introduction to the global geoid and global gravity.
Reading: Gravity Notes #3/#4, sections 8, 10 (Slides)
Homework: Problem Set #5 (due Monday, 2/9)

Lecture 13 - 2/3/2016
Topics: Reference ellipsoids; Brun's Formula for calculating geoid height; the gravity anomaly; spherical harmonic solution to Brun'sFormula
Reading: Gravity Notes #3/#4/#5, sections 9, 15
Additional resources: H&M 2-8, 2-11, 2-13

Lecture 14 - 2/5/2016
Topics: Global gravity models

Course Materials (Geomagnetism):

- Geomagnetism Notes #1 (pages 1-9)
- Geomagnetism Notes #2 (pages 10-14)
- Geomagnetism Notes #3 (pages 15-22)
- Geomagnetism Notes #4 (pages 22-38)
- Geomagnetism Notes #5 (pages 39-56)
- Geomagnetism Notes #6 (pages 57-64)
- Geomagnetism Notes #7 (pages 65-72)

Geomagnetism Lecture Contents:

Lecture 15 - 2/8/2016

Topics: Introduction to Geomagnetism
Reading: Geomagnetism notes, pages 1-9 (Slides)
Homework: Problem Set #6 (due 8am, Friday, 2/12)

Lecture 16 - 2/10/2016
Topics: Introduction to Geomagnetism – External Field
Reading: Today's Slides, animations provided as attachments to pdf file

Lecture 17 - 2/12/2016
Topics: Presentations on Magnetic Fields of Solar System Bodies
Reading: Today's Slides

Lecture 18 - 2/17/2016
Topics: 2 more presentations on Uranus and Neptune,
Begin Classical Electrodynamics in Geomagnetism
Reading: Geomagnetism notes #2, pages 10-14. Today's Slides

Lecture 19 - 2/19/2016
Topics: Classical Electrodynamics – the static case, and constitutive relations
Reading: Geomagnetism notes #2, pages 11-14. Today's Slides

Lecture 20 - 2/22/2016
Topics: Gauss' Theory of the Main Field: Internal/External Separation, Upward and Downward Continuation
Reading: Geomagnetism notes #3, pages 15-22. Today's Slides
Homework: Problem Set #7 (due Friday, 2/26)

Lecture 21 - 2/24/2016
Topics: Downward Continuation, Geomagnetic Field Models
Reading: Geomagnetism notes #3 and #4, pages 19-32. Here are the Slides.

Lecture 22 - 2/26/2016
Topics: The Geomagnetic Spectrum, Poloidal and toroidal fields
Reading: Geomagnetism notes #4, pages 32-38. Here are the Slides
Homework: Problem Set #8 (due Friday, 3/4)

Lecture 23 - 2/29/2016
Topics: Magnetohydrodynamics in Earth's Core
Reading: Here are the Slides

Lecture 24 - 3/2/2016
Topic: Secular variation
Reading: Here are the Slides

Lecture 25 - 3/4/2016
Topic: More Secular variation
Reading: Geomagnetism notes #5, page 39-56. Here are the Slides

Lecture 26 - 3/7/2016
Topic: A little dynamo theory
Reading: Geomagnetism notes #5, page 53-56. Here are the Slides
Solutions to Homework 7 are available here

Lecture 27 - 3/9/2016
Topic: Crustal Magnetization
Reading: Geomagnetism notes #6, page 57-64. Here are the Slides
Homework: Problem Set #9 (due Tuesday, 3/17)

Lecture 28 - 3/11/2016
Topic: Results from Lithospheric Modeling, Summary
Reading: Here are the Slides
Additional reading on Electromagnetic Induction in Geomagnetism notes #7

Supplementary Material:

Physical Geodesy
W. Heiskanen and H. Moritz
W.H. Freeman and Company, San Francisco, 1967
[On reserve in the Munk reading room.]
A thorough treatment of much of the gravity material covered in the class notes.

Potential Theory in Gravity and Magnetic Applications
Richard J. Blakely
Cambridge University Press, New York 1995
[available from ROGER (roger.ucsd.edu) in electronic format]
Another look at potential theory, specifically in the context of gravity and geomagnetism.

Treatise on Geophysics, Volume 3, Geodesy
Volume Editor: T. Herring. Editor-in-Chief: G. Schubert
Elsevier, Amsterdam, 2007
[available from ROGER (roger.ucsd.edu) in electronic format]
Technical summaries of many current topics in gravity research, written by the best in the field.

Foundations of Geomagnetism
G. Backus, R. Parker, C. Constable
Cambridge University Press, New York, 1996.

Treatise on Geophysics, Volume 5, Geomagnetism
Volume Editor: M. Kono. Editor-in-Chief: G. Schubert
Elsevier, Amsterdam, 2007
[available from ROGER (roger.ucsd.edu) in electronic format]
Technical summaries of many current topics in geomagnetic research.

Treatise on Geophysics, Volume 8, Core Dynamics
Volume Editor: P. Olson. Editor-in-Chief: G. Schubert
Elsevier, Amsterdam, 2007
[available from ROGER (roger.ucsd.edu) in electronic format]
Technical summaries of many current topics in core dynamics.

Space Science reviews volume on *Planetary Magnetism* at http://link.springer.com/journal/11214/152/1/page/1
And *Terrestrial Magnetism* http://link.springer.com/journal/11214/155/1/page/1