JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 106, NO. B7, PAGES 13,287–13,306, JULY 10, 2001

A model for sorted circles as self-organized patterns

M. A. Kessler, A. B. Murray,[1] and B. T. Werner

Complex Systems Laboratory, Cecil and Ida Green Institute of Geophysics and Planetary Physics
University of California-San Diego, La Jolla, California

B. Hallet

Quaternary Research Center and Department of Geological Sciences, University of Washington
Seattle, Washington

Abstract. Sorted circles emerge as self-organized patterns from a laterally uniform active layer that becomes laterally sorted as frost heave deforms the interface between a stone layer and an underlying soil layer. In a three-dimensional, cellular model of the active layer, cyclic freezing and thawing drives transport of stone and soil particles by (1) addition of ice particles representing soil expansion by frost heave, (2) removal of ice particles representing soil consolidation during thawing, (3) addition of void particles (a discrete abstraction of soil compressibility) representing soil expansion by water absorption, (4) removal of void particles representing compaction and desiccation of underlying soil by frost heave, (5) relaxation of surface morphology by soil creep and stone avalanche, and (6) vertical sorting of stones and soil by illuviation. These transport processes give rise to sorted circles, which are characterized by a mean spacing of 3.6 m, a 2.4 m wide soil domain surrounded by a 1.0 m wide, 0.3 m high annulus of stones, and a 750 year period of circulation in the soil domain, all consistent with measured characteristics of sorted circles in western Spitsbergen. In the model, instabilities on the stone-soil interface grow upward as soil plugs by drawing in soil from the surrounding subsurface soil layer; soil plugs develop into sorted circles as they contact the ground surface, simultaneously elevating an encircling annulus of stones. Sorted circles are dynamically maintained by circulation within the stone and soil domains. Initiation of soil plugs is driven by a positive feedback in which frost heave near the stone-soil interface pushes soil toward more compressible soil regions, where the soil layer is thicker. The lateral component of these frost-heave-induced displacements is not reversed during thaw because soil consolidation (as ice-rich soil melts and drains) and soil expansion (as desiccated and compacted soil hydrates) displace soil vertically. Further development of soil plugs and sorted circles is determined by an interplay between this positive feedback and amplitude dependent negative feedbacks that result from decoupling of the freezing front from the stone-soil interface. Parameters outside the range in which sorted circles form can result, for example, in stone islands and labyrinthine patterns. The initial wavelength of perturbations on the stone-soil interface is accurately predicted using a linear stability analysis, but increase in this wavelength through time reflects the nonlinearities that control the spacing of soil plugs and sorted circles, namely, interactions and mergers between neighboring forms.

1. Introduction

Freezing and thawing of Arctic surface soil results in sorted patterns of soil and stones, the most striking of which assume circular, polygonal, linear, and labyrinthine shapes. Although a range of mechanisms have been proposed for the origin of sorted patterned ground [*Washburn*, 1980, 1997], few of these hypotheses can be excluded, either because the proposed mechanisms have been only qualitatively described or because of a lack of discriminating measurements. Numerical models can be used to explore the consequences and consistency of these mechanisms. Here, a numerical model for one form of sorted patterned ground, sorted circles, is presented and investigated.

[1]Now at Division of Earth and Ocean Science/Center for Nonlinear and Complex Systems, Duke University, Durham, North Carolina.

Paper number 2001JB000279.
0148-0227/01/2001JB000279$09.00

1.1. Sorted Circles

A model for sorted circles should reproduce observed features and behaviors of the archetypical sorted circles found in western Spitsbergen [*Hallet and Prestrud*, 1986], the Canadian Arctic [*Washburn*, 1997], and Greenland [*Schmertmann and Taylor*, 1965]. In western Spitsbergen, sorted circles (Figure 1) typically consist of a 2-3 m wide cylinder of soil, the axis of which extends 1-1.5 m downward to the base of the active layer, the near-surface zone that freezes and thaws annually. The soil surface is domed upward as much as 0.1 m and is surrounded by a 0.5-1 m wide annulus of gravel, devoid of soil, that rises as much as 0.5 m above the soil domain [*Hallet and Prestrud*, 1986; *Hallet et al.*, 1988]. Well-developed sorted circles in western Spitsbergen did not detectably change form over a decade of measurements [*Hallet*, 1998]. Coherent surface soil motion of ~0.01 m/yr outward from the center of the soil domain has been recorded in Greenland [*Schmertmann and Taylor*, 1965] and in western Spitsbergen [*Hallet and Prestrud*, 1986], as has stone motion of comparable velocity down the inner slope of the gravel annulus [*Hallet and Prestrud*, 1986]. Subsurface soil transport toward the center of the soil domain has been inferred from surface morphology, surface soil motion, and the assumption that soil volume is conserved [*Hallet and Prestrud*, 1986], but no direct measurements are available. This inference is supported by tilt measurements from within a sorted circle soil domain that record substantial yearly subsurface rotational motion [*Hallet*, 1998]. A soil domain circulation period of 500 years has been estimated for a sorted circle in western Spitsbergen with a 3 m wide, 1 m thick soil domain [*Hallet et al.*, 1988].

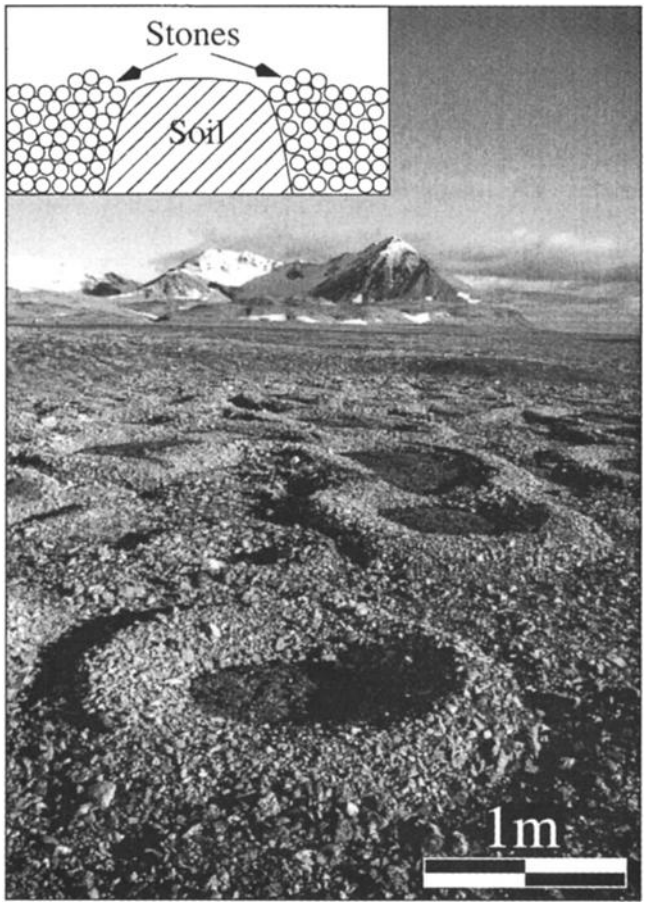

Figure 1. Sorted circles from the Kvadehuke Peninsula in western Spitsbergen. Inset shows schematic cross section of a sorted circle (after Prestrud, 1987).

Cyclic freezing and thawing drives processes that transport stones, soil, and water in the active layer, resulting in the development of sorted circles. During winter, a freezing front descends from the ground surface with a velocity dependent on the thermal properties and configuration of stones and soil in the active layer. Measurements of temperature within a sorted circle during freezing indicate faster freezing front propagation in the stone domain than in the soil domain [*Schmertmann and Taylor*, 1965]. Freezing of fine-grained soils impels moisture migration to the freezing front, resulting in expansion where water freezes, termed frost heave, and contraction where soils are desiccated by water withdrawal [*Taber*, 1929]. During thaw, ice-rich soils compact by expelling excess water and desiccated soils expand by absorbing water. Soil expansion is also caused by frost heave in still frozen underlying soils as water released by melting percolates downward [*Mackay*, 1980]. Percolating water entrains soil particles and transports them downward through pores between stones to the base of stone regions with unfilled pores, a process termed soil illuviation [*Corte*, 1966; *Forman and Miller*, 1984; *Washburn*, 1997]. Surface gradients generated by the subsurface redistribution of stones and soil by freezing and thawing cause downslope soil creep [*Hallet and Prestrud*, 1986; *Hallet et al.*, 1988]. Sorting in the active layer arises from the dependence of these transport processes on the thermal and hydrological differences between stones and soil, resulting in laterally and vertically nonuniform stone and soil displacements [*Taber*, 1929; *Corte*, 1961; *Anderson*, 1988].

Several environmental conditions are common to locations where sorted circles are found. First, the surface layer experiences cycles of annual freezing and thawing. Second, soil with volumetric fractions of frost-susceptible fine material in the range 12-42% is present in the active layer for all types of sorted patterned ground [*Goldthwait*, 1976]. Third, water remains near the ground surface during freezing because underlying permafrost inhibits drainage and the high-fine-content soils retain water. Fourth, air temperatures only moderately below 0°C during freezing cause the active layer to freeze slowly (over a period of several weeks to months), resulting in substantial frost heave [*Hallet and Prestrud*, 1986; *Hallet*, 1998].

1.2. Current Problem State

Dozens of hypotheses, relying on 19 basic mechanisms [*Washburn*, 1956, 1997], have been proposed for specific

types and aspects of sorted patterned ground. Proposed mechanisms for sorted circles have primarily addressed three stages in their development: pattern initiation, accumulation of soil into a central soil domain, and the dynamics of well-developed forms.

In one model for pattern initiation, inversion in the density of interstitial water from the base of the active layer (at 0°C) to the ground surface (at 4°C) during thawing results in a cellular pattern of thermal convection. Down welling warm water melts the upper surface of the permafrost, modifying the interface between the active layer and the underlying permafrost. The resulting undulating interface biases active layer transport processes to produce sorted circles with the same pattern as the convection cells. The ratio of sorted circle width to active layer depth predicted from a linear stability analysis is consistent with measurements [*Ray et al.*, 1983; *Gleason et al.*, 1986; *Krantz*, 1990]. However, this model relies on coherent convection of water through soil pores, which has been documented in aquifers and snow layers [*Combarnous and Bories*, 1975] but not within the highly heterogeneous soil of the active layer.

Because a layer of soil commonly underlies a layer of stones in areas where sorted circles are forming [*Elton*, 1927; *Corte*, 1961; *Washburn*, 1956, 1969, 1997], a layered configuration often is hypothesized to constitute an initial condition for sorted circle formation. Layering can arise directly from sorting inherent in the depositional processes that produced the soil (e.g., beach deposits and differential weathering) or from active layer processes, such as illuviation and frost-heave-driven stone uplift, operating on an initially mixed soil [*Washburn*, 1997]. Mechanisms proposed for the accumulation of the soil layer into a central soil domain include spatially varying frost heave [*Washburn*, 1956; *Nicholson*, 1976; *Van Vliet-Lanoe*, 1991; *Washburn*, 1997] and a density-driven instability of the stone-soil interface [*Van Vliet-Lanoe*, 1991; *Washburn*, 1997]. These mechanisms are related to sorted circle formation with the assumption that diapiric soil plugs are a precursor to well-developed sorted circles.

One mechanism proposed for the maintenance of fully developed sorted circles is convection of soil owing to decreasing soil density with depth. This density inversion is hypothesized to be caused by compaction during thaw that starts at the ground surface and descends through the active layer (*Mortensen* [1932] referenced by *Washburn* [1956] [see also *Hallet and Prestrud*, 1986]). However, measured soil density gradients are below the predicted value necessary to initiate soil convection [*Hallet and Waddington*, 1991]. In a second mechanism, frost heave at inclined freezing fronts maintains sorted circles because stones move upward toward a descending freezing front but fine-grained particles are expelled downward away from the freezing front [*Nicholson*, 1976]. In a third mechanism, first proposed for earth hummocks [*Mackay*, 1979, 1980] and later extended to sorted circles [*Washburn*, 1997], frost heave at a descending freezing front desiccates and compacts the soil domain, followed by vertical soil expansion during thaw and inward soil transport along a concave upward thaw front. The second and third mechanisms are incomplete in that they postulate but do not produce an initial freezing front or thawing front inclination [*Washburn*, 1997].

Many of the proposed mechanisms have not been modeled physically, analytically, or numerically. None of the models or mechanisms treat sorted circles from inception to fully developed forms. The predictive capacity of individual models is limited because of the small range of behavior to which they are applicable and because the consistency between models for different aspects of sorted circles has not been explored.

1.3. New Approach

Here we present a numerical model for sorted circle initiation, formation, and maintenance based on a set of hypothesized simplifications of the interactions between lateral sorting, freezing front inclination, and transport of stones and soil in an active layer undergoing repeated freezing and thawing. The approach underlying this model relies on two key assumptions. First, sorted circles emerge owing to self-organization, the development of global order from local interactions [*Nicolis and Prigogine*, 1977], implying that mechanisms in the model need not reflect the form or scale of the patterns, which emerge spontaneously. The initiation and stabilization of self-organized patterns are dependent only on the general nature of positive and negative feedbacks acting locally. Second, it is assumed that many of the details of heat flow and stone, soil and water transport processes need not be included in a model for sorted circles. This insensitivity to detail stems from a robust property of nonlinear, dissipative systems: the loss of information associated with dissipation results in only some aspects of fast, small-scale processes contributing to the dynamics of slower-evolving, larger-scale patterns that emerge [*Werner*, 1995, 1999].

2. Conceptual Model

In our conceptual model, sorted circles evolve within an active layer that is forced by an annual cycle of freezing and thawing. The ground surface and the surface of the underlying permafrost constitute the boundaries of the active layer. Heat flow through the ground surface and flow of water into or out of the active layer are the primary interactions between the active layer system and the external environment. Active layer processes act on stones and soil differently because of their differing thermal and hydrological properties. Soil retains water, releases latent heat during freezing, enables frost heave, compacts when desiccated or compressed, creeps down low slopes, sifts downward through the pores between stones, and expands upon absorption of water. Stones exhibit none of these behaviors. Sorted circles

emerge because of feedbacks between the configuration of stones and soil in the active layer and the transport resulting from active layer processes acting on that configuration.

2.1. Processes

Frost heave is the primary transport process in our conceptual model for sorted circle formation. Soil expansion at the freezing front, by ice growth, and contraction of the underlying unfrozen soil, by compaction and desiccation, result in subsurface transport away from the freezing front. The freezing front moves downward from the ground surface at a rate determined by heat conduction and the release of latent heat when water freezes. Where stones and soil form distinct domains separated by a well-defined interface, with stones overlying soil, the morphology of the freezing front approximates the stone-soil interface because the freezing front moves rapidly through dry stones but more slowly through wet soil. Frost heave pushes upward, displacing stones and soil, and deforming the ground surface. In addition, frost heave and frost-heave-induced desiccation displace soil downward, compacting soil well below the stone-soil interface. Frost heave primarily induces soil expansion near the top of the soil layer because of the limited supply of water available for ice formation, limited soil compressibility, and increasing overburden with depth.

Gravity drives three relaxation phenomena: compaction during thaw, soil illuviation, and downslope surface transport. Thawing, ice-rich soils expel water and compact vertically under the overburden. A distinct interface between stone and soil domains is maintained by illuviation, soil sifting downward through the pores between stones. On the ground surface, soil creeps downslope, and gradients within the stone domain relax (stone particles avalanche) when the surface slope exceeds the angle of repose.

During thawing, compressed and desiccated soil expands by absorption of downward percolating water. The direction of expansion is vertical because stresses caused by lateral confinement are assumed to exceed those from the overburden. Expansion of soil by water absorption during thaw reverses (in magnitude, not direction) the compaction of soil by frost heave during freezing because the soil is assumed to have undergone many loading and unloading (freeze-thaw) cycles with similar maximum stresses, which bring a soil into a steady state with equal magnitudes of compression and expansion [*Lambe and Whitman*, 1969, p. 321].

2.2. Positive Feedback Mechanism

In the conceptual model, the initial configuration of the active layer is a uniform layer of mixed stones and soil underlying a uniform layer of stones devoid of soil. Sorted circles initiate as an instability in the interface between the stone and soil layers. Small positive perturbations in the stone-soil interface are more compressible because of the greater volume of soil beneath them. After compressing soil locally, frost heave near the stone-soil interface transports soil into nearby perturbations where unfrozen compressible soil remains. The net effect of soil transport by frost heave is to increase the amount of soil within those positive soil perturbations (Figure 2). Through repeated freezing and thawing, perturbations in the interface grow upward because of the asymmetry between soil compaction during freezing (Figure 2a), which has a lateral component, and vertical soil relaxation and expansion during thawing (Figures 2b and 2c). This positive feedback initiates the pattern and leads to the accumulation of soil and the formation of soil plugs. We hypothesize that these same processes underlying pattern initiation, combined with surface relaxation, can lead to the development and maintenance of sorted circles. Investigation of this hypothesis requires a numerical model implementing these processes.

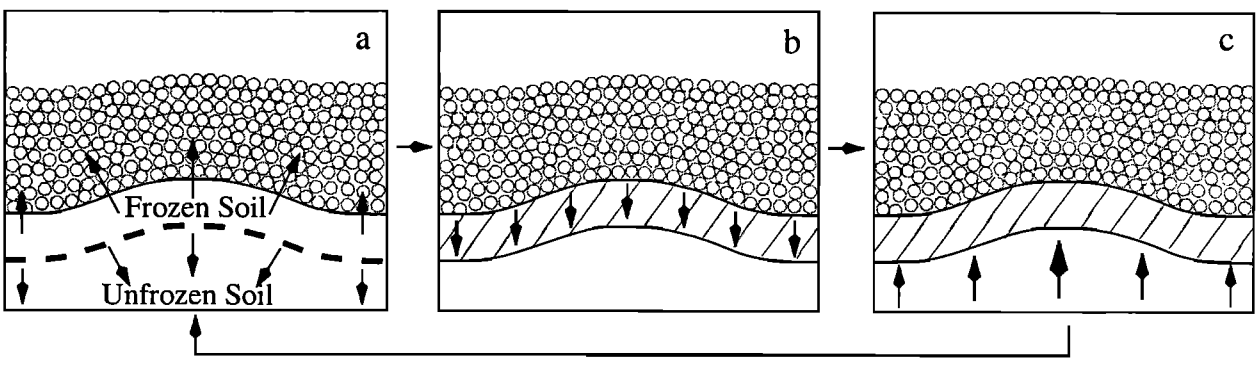

Figure 2. Conceptual model: cross sections indicating transport during a freeze-thaw cycle resulting in positive feedback mechanism for soil accumulation and destabilization of stone-soil interface. During freezing, (a) frost heave drives material away from the freezing front (dashed line), outward toward the ground surface and inward toward unfrozen soil. During thawing, (b) ice-rich soil near the stone-soil interface expels water and compacts vertically, while (c) desiccated and compacted soil expands vertically by absorbing water.

3. Numerical Model

This conceptual model for sorted circles is encoded into a three-dimensional cellular computer model that simulates the annual freeze-thaw cycle of an active layer. The simulated active layer is bounded by a rigid plane lower surface, a mechanically free upper surface, and periodic lateral boundaries.

Four types of particles occupy the cells representing the active layer: stone, soil, ice, and void. These particles move as indivisible units. Ice particles and void particles occupy an entire cell. Stone and soil particles each fill 50% of the volume of a cell. Because small grain-size soil particles can fit in the pore space of cells containing large grain-size stone particles, a cell can contain either one stone particle, one stone particle and one soil particle, or two soil particles, but not two stone particles. Stone particles represent either one or many stones that move as a unit. Void particles do not represent pores in soil; rather, void particles are a discrete representation of soil compressibility (given stresses from a typical freeze-thaw cycle). Cells containing one stone particle and one soil particle will be termed soil cells because in the simulations presented here (excepting section 4.4), all cells containing a soil particle also contain a stone particle (i.e., no cells contain two soil particles). The corresponding volumetric fraction of stones in soil domains, 0.5, is comparable to the value measured from a sorted circle in western Spitsbergen, 0.31 [*Prestrud*, 1987]. The numbers of stone and soil particles are conserved. Cells are half as high as they are wide to enhance vertical resolution while maintaining computational performance.

The thermal and hydrological properties of soil particles differ from those of stone particles. During freezing, the water content of soil particles, which determines the latent heat but does not constrain ice particle formation, is assumed to be a fraction w of the soil particle volume. Stone particles do not contain water. During freezing, ice particles can form in soil cells but not stone cells. Soil particles sift through the pore space of stone cells and creep downslope at the ground surface. Stone particles at the ground surface avalanche only when gradients exceed a specified angle of repose.

Void particles and ice particles are created and destroyed during the yearly freeze-thaw cycle. Void particles form within soil domains and render the simulated active layer compressible. The volume of void particles divided by the volume of soil particles (the volume of a soil particle is half the volume of a cell) is equivalent to the soil compressibility C. The formation of ice particles simulates the expansion of soil by frost heave.

3.1. Freeze-Thaw Cycle

In the numerical model, a freeze-thaw cycle is represented by a sequence of steps corresponding to the physical processes of the conceptual model.

3.1.1. Freezing and frost heave. As the active layer freezes, the time-dependent temperature in each cell, the descent of the freezing front ($\equiv 0°$C isotherm), the formation of ice particles, and the resulting stone and soil displacements are simulated (Figure 3a). An initial temperature profile, representative of late summer, is imposed, increasing linearly from T_b at the base of the active layer up to T_g at the ground surface. The temperature of unoccupied cells above cells representing the ground surface is held at T_a for the freezing step, representative of the average air temperature during freezing. The temperature within each cell in the active layer evolves because of heat diffusion and release of latent heat. The temperature change is calculated by time stepping the DuFort-Frankel finite difference approximation of the three-dimensional heat diffusion equation [*DuFort and Frankel*, 1953; *Nogotov*, 1978], with a correction for latent heat release after each time step; the change in temperature of a cell at 0°C is converted to an equivalent release of latent heat and reduction in the unfrozen water content. When the unfrozen water reservoir of a cell at the freezing front is exhausted, the local freezing front descends to the cell below.

When the freezing front exits a cell containing a soil particle ("t" in Figure 3a), a test for formation of an ice particle in that cell is conducted. An ice particle displaces the particles from the cell in which it forms either toward a void cell in the unfrozen soil or toward the ground surface. The probability of an ice particle forming is the greater of two probabilities: that of displacing particles toward the nearest void cell or that of displacing particles toward the ground surface. These probabilities are assumed to decrease linearly with distance:

$$P_{\text{surf}} = \max(0, 1 - d_{\text{surf}}/d_s) \quad (1a)$$

$$P_{\text{void}} = \max(0, 1 - d_{\text{void}}/d_v), \quad (1b)$$

where d_{surf} is the distance to the ground surface, d_{void} is the distance to the void cell, parameters d_s and d_v are the distances at which P_{surf} and P_{void} are zero, and $\max(A, B)$ is the greater of A and B. Displacements toward the ground surface follow a path that is a weighted average of the unit vector normal to the freezing front at the ice particle and the unit vector pointing from each displaced cell in the path to the nearest empty cell above the ground surface. The weights are proportional to the inverse distances from the displaced cell to the ice particle and from the displaced cell to the ground surface, respectively. This path of displacements starts out perpendicular to the freezing front, then bends toward the ground surface. Displacements toward a void particle follow a straight line from the ice particle to the void particle. This algorithm for ice formation and particle displacement abstracts the complicated processes of frost heave, compaction, desiccation, granular shear,

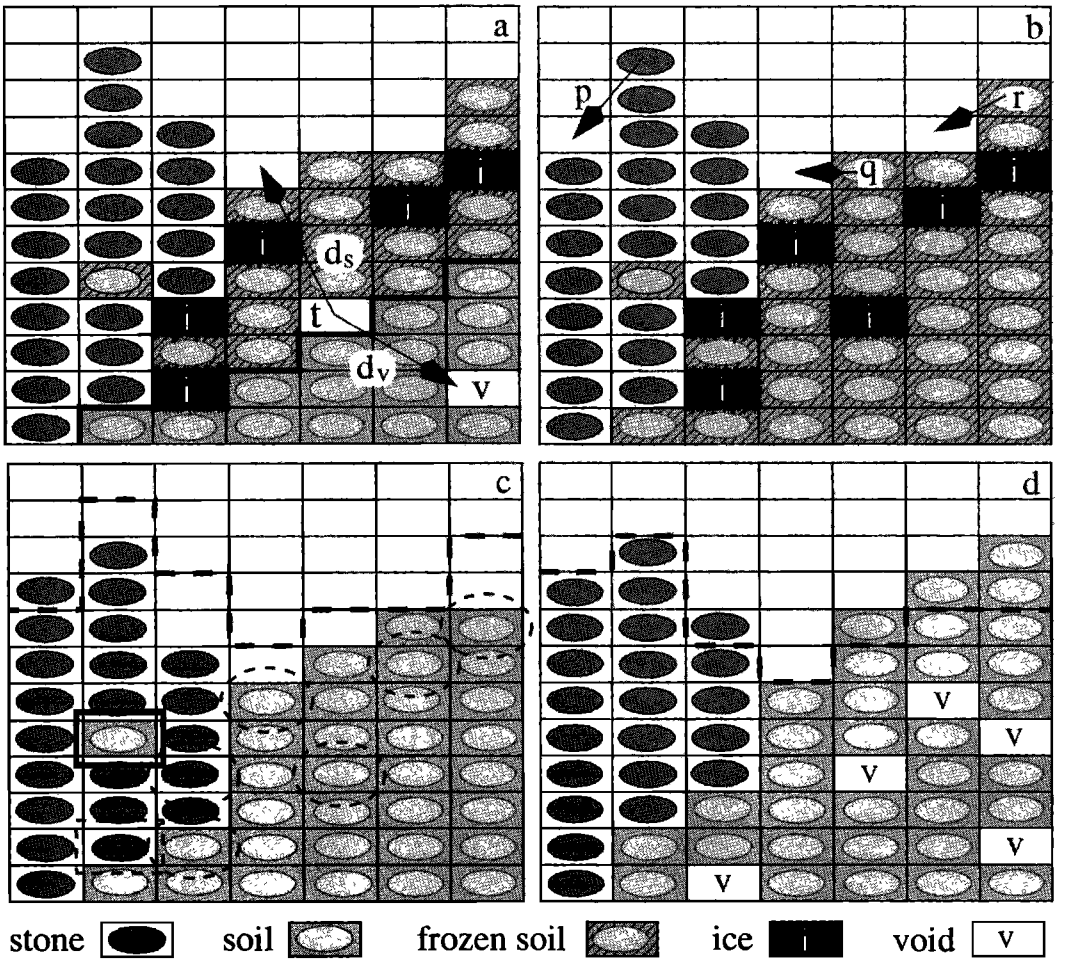

Figure 3. Cross sections through a schematic of the numerical model for several stages in the simulated freeze-thaw cycle. Stone, unfrozen soil, frozen soil, ice, and void particles are as indicated. (a) Freezing and frost heave stage. The freezing front (thick solid line) migrates from the surface downward by heat diffusion modulated by release of latent heat from soil particles. An ice particle can be inserted at the location labeled "t" on the freezing front, depending on the probability of displacing the particles at t toward the ground surface or toward a void cell, equation (1). (b) Surface morphology relaxation stage. Stone particle, labeled "p", moves down a slope exceeding the angle of repose. Soil particles, labeled "q" and "r", move downslope with probabilities proportional to slope. (c) Compaction during thaw and soil illuviation stage. Ice particles are removed (dashed ovals), and the columns of stone and soil particles above the vacated cells are shifted downward. Dashed line indicates the surface after freezing but before surface motion. Soil particle in the stone domain (solid rectangle) moves downward to the lowest stone cell in the column (dashed rectangle). (d) Soil expansion stage. Void particles are added with a probability equal to the local volume fraction of soil until a specified compressibility C is reached. Dashed line indicates the surface before adding void particles.

viscoelastic deformation, and water migration in partially frozen soils.

3.1.2. Surface morphology relaxation. During thawing, downslope surface motion relaxes surface morphology (Figure 3b). Two-dimensional diffusion of elevation, a continuum approximation that models surface soil flux proportional to slope [e.g., *Jyotsna and Haff*, 1997], is incorporated in the model by moving particles at the ground surface to the lowest of the eight surrounding cells with a probability $\Delta V/V_o$, where V_o is the volume of a cell and ΔV is the volume that would diffuse during a time interval Δt given a difference in elevation between the two cells of Δh:

$$\Delta V = K_{\text{surf}} \frac{\Delta h}{\Delta x} \Delta t, \tag{2}$$

where K_{surf} is a diffusion constant and Δx is the distance between cell centers. The time step Δt is chosen such that at most one particle per column is displaced in a single time step. Stone particles resting on stone particles move downslope only if the surface elevation difference exceeds two cells, which is equivalent to an angle of repose of 45°.

3.1.3. Compaction during thaw and soil illu-

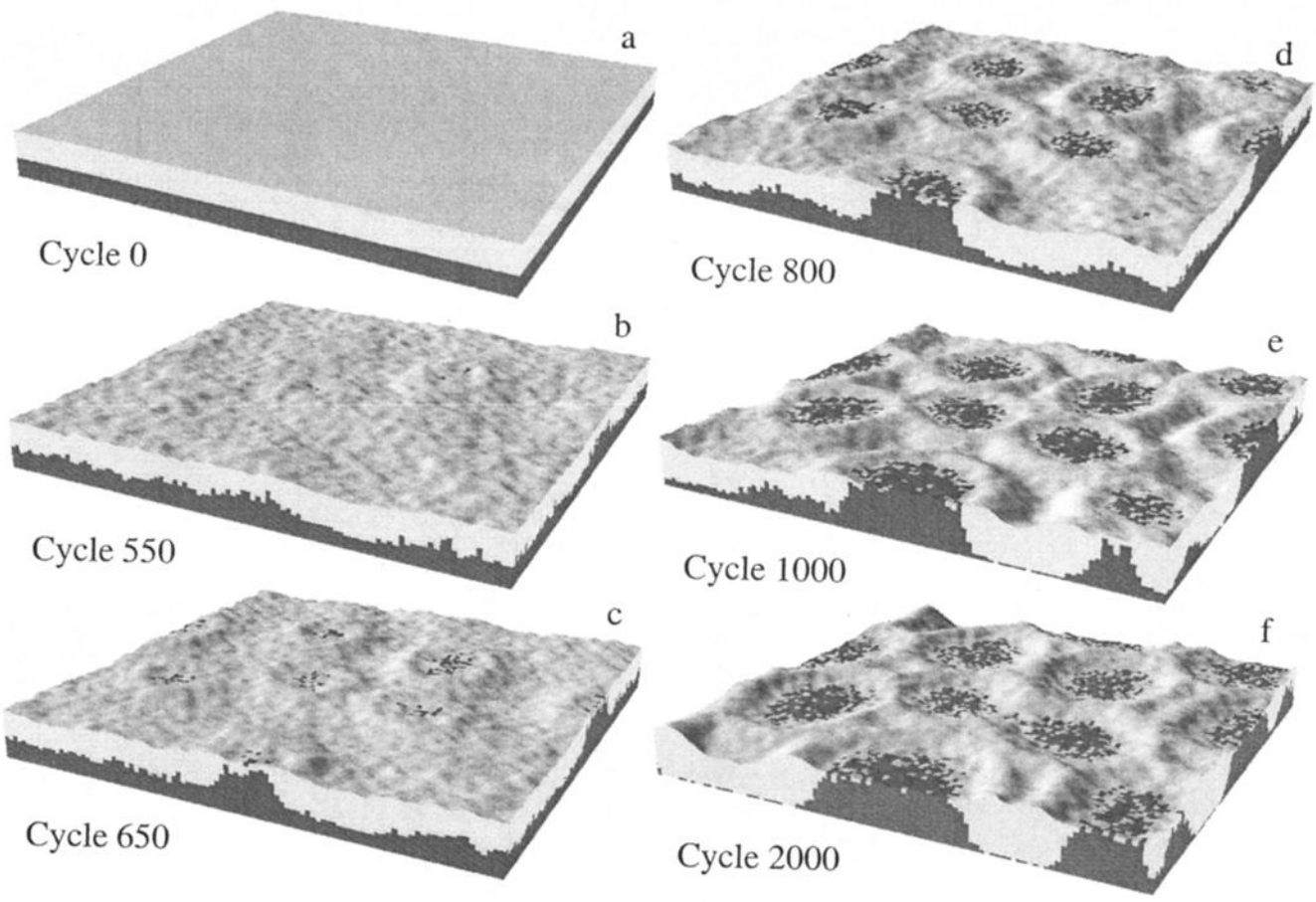

Figure 4. Sorted circle formation and development in a 10 m × 10 m simulation with reference model parameters. (a) Initial configuration has a 0.6 m thick stone layer (light shading) overlying a 0.4 m thick soil layer (dark shading). (b) Perturbations in the stone-soil interface first reach the surface at ~550 freeze-thaw cycles. (c) Frost heave at the stone-soil interface elevates a surrounding annulus of stones. (d) Frost heave drives relatively rapid radial expansion. (e) The underlying soil layer has been largely depleted. (f) Over long time scales relatively slow radial expansion, interaction between adjacent forms, and maintenance of fully developed forms continues.

rounded by a 1 ± 0.2 m wide annulus of stones elevated ~0.3 m above the soil domain (Figure 4f). Sorted circles are spaced 3.6 ± 0.6 m apart in the reference model.

Mean surface motion is outward from the center of the soil domain to its intersection with the stone domain (zone 1, Figure 8) and inward from the crest of the stone annulus to its intersection with the soil domain (zone 2, Figure 8). The small outward velocity beyond the crest of the stone annulus (zone 3, Figure 8) indicates continued radial growth. A 12 year record of the position of surface markers on a sorted circle in western Spitsbergen shows a similar pattern of surface motion. Subsurface recirculation of soil from the periphery of the soil domain inward (Figure 9) maintains the convex surface despite the high outward velocity in zone 1. The strong velocity gradients near the stone-soil interface and roughly uniform upward motion in the central region of the soil domain are similar to inferred sorted circle circulation patterns [*Pissart*, 1990]. Al-

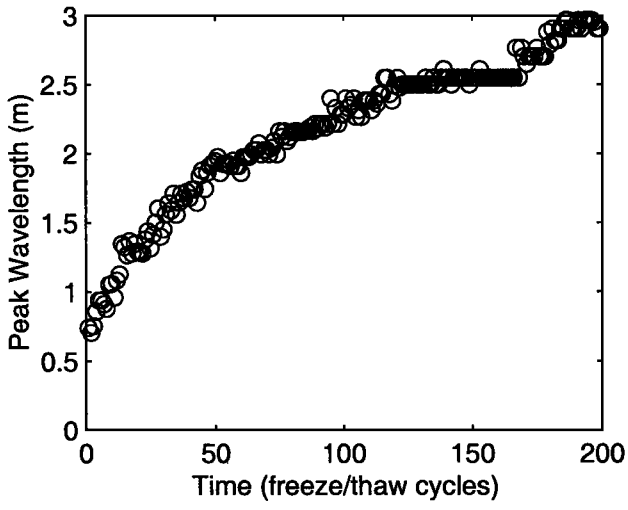

Figure 5. Peak wavelength (from angle-averaged, radial power spectrum) of perturbations on the stone-soil interface versus number of freeze-thaw cycles.

viation. During thaw, ice-rich soil compacts, and soil particles sift downward through the pore space of underlying stone cells (Figure 3c). First, ice particles are removed and the overlying column of particles is lowered one cell. Second, soil particles located above cells containing only a stone particle move downward to fill the lowest stone cell, thereby transforming it into a soil cell.

3.1.4. Soil expansion. During thaw, desiccated soil hydrates and expands (Figure 3d). Soil expansion is simulated by adding void particles to the soil domains until a prespecified soil compressibility (total volume of void particles divided by total volume of soil particles), C, is achieved. The probability of inserting a void particle in a cell is C times the volumetric fraction of soil particles in a $3 \times 3 \times 3$ cell neighborhood. To make room for a void particle, the column of particles above the cell in which it will be inserted is shifted up one cell. At the conclusion of the thaw stage, cells are assigned an unfrozen water content equal to the water fraction w times the volume of soil in that cell (half the volume of a cell for soil cells and zero for stone cells).

3.2. Reference Model

A set of parameters characterizing a reference model was chosen, in part, based on measurements from sorted circles in western Spitsbergen (Table 1). The active layer is 1 m thick [*Hallet and Prestrud*, 1986], with an initial configuration of a 0.6 m thick layer of stone cells, $h_{\text{stone}} = 0.6$ m, overlying a 0.4 m thick layer of soil cells, $h_{\text{soil}} = 0.4$ m. This ratio of stone to soil volume approximately corresponds to that of a 3 m wide soil cylinder surrounded by a 1 m wide stone annulus, within the observed range of sorted circles in western Spitsbergen [*Hallet and Prestrud*, 1986]. Before freezing, the soil compressibility C is 0.05, consistent with expansion measured in high clay content soils [*Lambe and Whitman*, 1969, p. 323]. The volumetric fraction of water in soil particles w is 0.1. This value, which is lower than observed in natural soils of this type, was chosen because higher values of w have little effect on model results (e.g., Figure 12e) but result in significantly increased simulation time. The linear temperature profile before freezing is bounded by $T_b = 0°$C at the base of the active layer and $T_g = 5°$C at the ground surface (mean summer air temperature) [*Putkonen*, 1998]. The air temperature during freezing is assumed to be the annual mean, $T_a = -5°$C [*Putkonen*, 1998]. Assuming the large difference in latent heat capacity between stones and soil overwhelms small differences in heat conductivity, a typical thermal diffusivity of $k_{\text{heat}} = 10^{-6}$ m^2/s is used for all cells [*Williams and Smith*, 1989]. The proportionality factor between surface slope and soil flux for surface creep is $K_{\text{surf}} = 5 \times 10^{-3}$ m^2/yr, yielding a surface velocity comparable to the measured value ~0.01 m/yr on a ~1 m radius sorted circle with a maximum elevation difference of ~0.1 m [*Hallet et al.*, 1988], assuming this velocity decreases linearly to zero at roughly the same depth as the maximum elevation difference, 0.1 m. Little information regarding the length scales characterizing subsurface displacement in the active layer is available. In the reference model, the distances at which the probabilities for displacing particles to the ground surface or to a void cell fall to zero are set to $d_s = d_v = 0.6$ m, a significant fraction of the active layer depth. Cells measure 0.1 m × 0.1 m × 0.05 m. A reference model simulation is 200 × 200 cells or 20 m × 20 m in size.

Table 1. Reference Model Parameters

Parameter	Value
d_s, max. dist. heave to surface	0.6 m
d_v, max. dist. heave to void	0.6 m
h_{stone}, stone layer thickness	0.6 m
h_{soil}, soil layer thickness	0.4 m
w, vol. fraction water in soil	0.1
C, soil compressibility	0.05
T_b, active layer base temp.	0°C
T_g, surface temp. before freezing	5°C
T_a, air temp. during freezing	−5°C
k_{heat}, heat diffusion const.	1×10^{-6} m^2/s
K_{surf}, height diffusion const.	5×10^{-3} m^2/yr

4. Results

4.1. Reference Model

In the reference model, patterns of stones and soil resembling sorted circles form and evolve over ~500-2000 freeze-thaw cycles (Figure 4; the simulation shown is 10 m × 10 m for illustrative purposes). The stages of sorted circle development, including initiation, soil accumulation, and maintenance, are steps in a continuous process that can be followed for any sorted circle in the reference model; however, the timing varies between individual sorted circles.

From an initially horizontal stone-soil interface (Figure 4a), coherent distortions of the interface much larger than the cell size develop in the first few freeze-thaw cycles. Between 1 and 200 freeze-thaw cycles, the peak wavelength in these perturbations, measured from a radial power spectrum (a two-dimensional power spectrum that has been averaged over angle for each radial wavenumber), increases from ~0.7 to ~3 m (Figure 5). Between 200 and 600 freeze-thaw cycles, discrete soil plugs emerge and expand toward the surface by collecting soil from the surrounding soil layer and through mergers with neighboring soil plugs (Figures 4b and 6). Between 500 and 700 freeze-thaw cycles, sorted circles emerge (Figures 4b and 4c). After soil domains contact the ground surface, they continue to expand rapidly over ~500 freeze-thaw cycles (Figures 4b-4e and 7) until the underlying soil layer is depleted (Figure 4e). In well-developed forms a 2.4±0.5 m diameter circular soil domain extends to the base of the active layer and is sur-

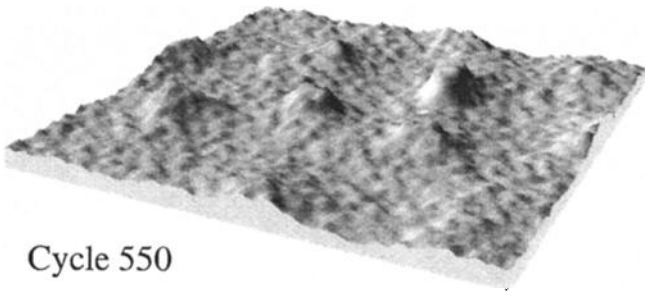

Figure 6. The stone-soil interface in the reference model after 550 freeze-thaw cycles, illustrating soil plugs. Overlying stone layer in Figure 4b has been removed.

though the streamlines of circulation in the soil domain are irregular (Figure 9), the mean period of circulation can be approximated assuming a circular path with a radius half the active layer depth, 0.5 ± 0.2 m, and a mean soil velocity of 0.0042 ± 0.001 m/yr, which is averaged over 10 freeze-thaw cycles starting with cycle 1500 (the velocity field is calculated from 200 simulations starting with the same configuration of stones and soil but using different sequences of random numbers). The mean period of circulation, 750 ± 350 years, is comparable to the 500 year period inferred for sorted circles in western Spitsbergen [*Hallet and Prestrud*, 1986].

4.2. Sensitivity to Parameters

Qualitatively, whether sorted circles form does not depend on most of the model parameters. However, quantitative characteristics of sorted circles, including spacing, size, and formation time, are sensitive to some parameters. The effects of varying simulation parameters one at a time from their reference model values on the following characteristics of sorted circles are discussed: (1) initial spacing between soil plugs and between sorted circles, (2) time for sorted circles to form, (3) mean circulation velocity of the soil domain, and (4) sorted circle formation criteria.

4.2.1. Initial spacing. In the reference model, soil plugs form as isolated features in a manner that depends primarily on the initial thicknesses of the stone and soil layers, h_{stone} and h_{soil}, and on the distance at which the probability of an ice particle displacing soil to a void cell falls to zero, d_v. If the soil layer thickness is less than ~40% of the active layer thickness, the initial spacing between sorted circles equals the spacing between soil plugs. If the thickness of the soil layer is greater then ~40% of active layer thickness, soil plugs form faster and closer together, evolving into circular, elongate, and labyrinthine patterns (Figure 10), primarily through mergers between neighboring soil domains. In this case, spacing depends in a complicated manner on parameters.

The initial spacing between soil plugs, estimated from the peak of the radial power spectrum of the stone-soil interface height, increases as d_v increases (Figure 11a). Spacing also varies weakly with the inverse of the water content of soil w, decreasing from 3.3 m to 2.5 m as w increases from 0.02 to 0.3. A similar dependence holds for T_a, with spacing decreasing as T_a increases. As the thickness of the initial soil layer h_{soil} increases, the spacing between soil plugs decreases (Figure 11b) because the minimum volume of soil required for soil plugs to rise to the surface can be acquired from a smaller area. Spacing does not fall below ~2 m because soil plugs separated by < 2 m readily merge.

4.2.2. Formation time. The time required for sorted circles to form, taken here to be the number of freeze-thaw cycles before a soil plug contacts the ground surface, depends on the rate at which soil accumulates into plugs and the volume of soil required for a soil plug to reach the ground surface.

The primary parameters determining soil accumulation rate are the compressibility C and h_{soil} (Figures 12a and 12b). As soil compressibility increases, the rate of soil accumulation is amplified by increasing the number of subsurface transport events per freeze-thaw cycle. An increase in h_{soil} also results in an increase in the number of subsurface transport events per freeze-thaw cycle and a consequent decrease in the time required to form sorted circles.

The volume of soil plugs contacting the surface is determined by the parameters d_v, h_{stone}, w, and T_a. The mean distance that soil is transported in a freeze-thaw cycle increases with d_v, thereby increasing the soil collection rate and tending to decrease sorted circle formation time. However, if $d_v < \sim 0.8$ m, the spacing between soil plugs (Figure 11a) and the initial soil plug volume both increase with increasing d_v sufficiently fast to overwhelm the increasing soil collection rate, resulting in increasing formation time (Figure 12c). If $d_v > \sim 0.8$ m, the spacing between developing soil plugs

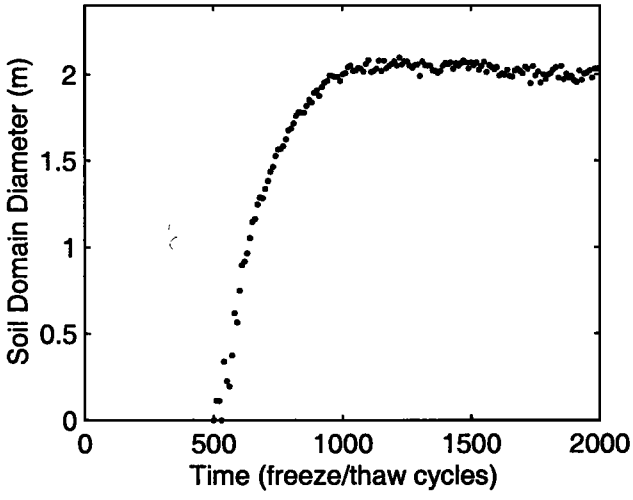

Figure 7. Soil domain diameter for a single sorted circle in the reference model versus number of freeze-thaw cycles. Diameter is calculated from the number of soil cells at the surface, assuming a circular soil domain.

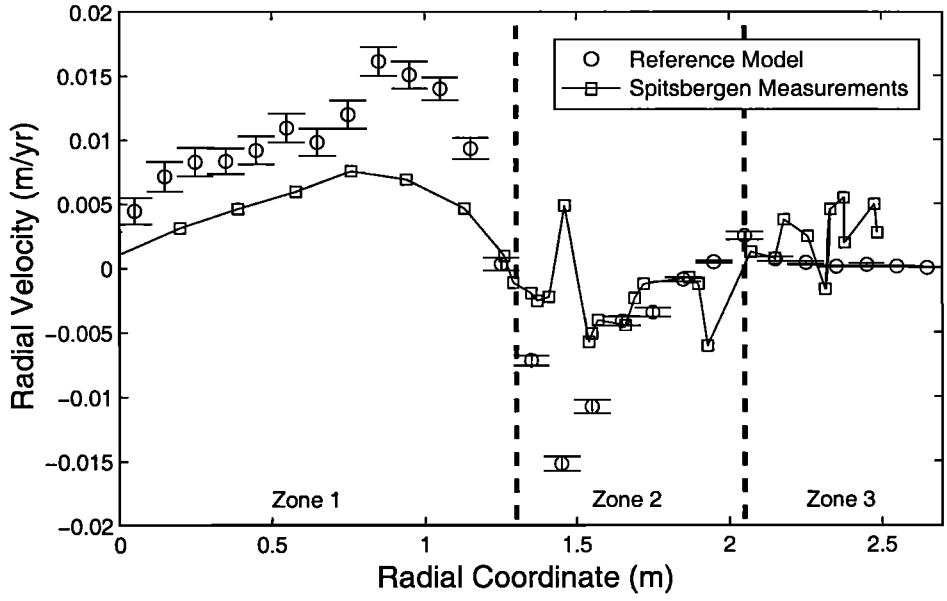

Figure 8. Mean surface velocity on a well-developed sorted circle in the reference model and mean surface velocity on a well-developed sorted circle in western Spitsbergen based on sequential measurements during a 12 year period starting in 1985. Markers, consisting of both marked stones and pegs vertically implanted within the upper decimeter of soil, were measured manually (see *Hallet and Prestrud* [1986] for discussion of the measurement technique and analysis of errors (~2 mm)). Reference model velocity field is an average of 200 simulations, with different sequences of random numbers, of a single sorted circle during the 10 year period from cycle 1501 to cycle 1510; 1σ error bars reflect the variation in these simulations. Positive velocities indicate motion away from the soil domain center. In the model, surface motion in zone 1 (soil domain) is away from the center of the sorted circle and decreases to zero at the stone-soil intersection. In zone 2 (inner slope of stone domain), surface motion is toward the center of the sorted circle. In zone 3 (outer slope of stone domain), surface motion is away from the center of the sorted circle, indicating radial growth. A similar pattern of displacements was measured in western Spitsbergen.

is determined by interactions and mergers that are less strongly correlated with d_v; in this case, the increasing mean soil displacement distance per freeze-thaw cycle with increasing d_v decreases formation time. Sorted circle formation time increases with increasing h_{stone} (Figure 12d) because soil plugs require more time to penetrate a thicker stone layer.

Formation time is sensitive to w and T_a through their effects on the amount of ice in soil plugs relative to surrounding regions. Greater ice formation in soil plugs displaces soil into the surrounding regions, suppressing soil plug growth and selecting for larger spacing and volume of soil plugs contacting the ground surface. Increasing w increases the stabilizing effect of latent heat (coupling the freezing front to the stone-soil interface), resulting in a more uniform distribution of ice on the stone-soil interface, less suppression of soil plug growth, smaller initial volumes, and shorter formation times for

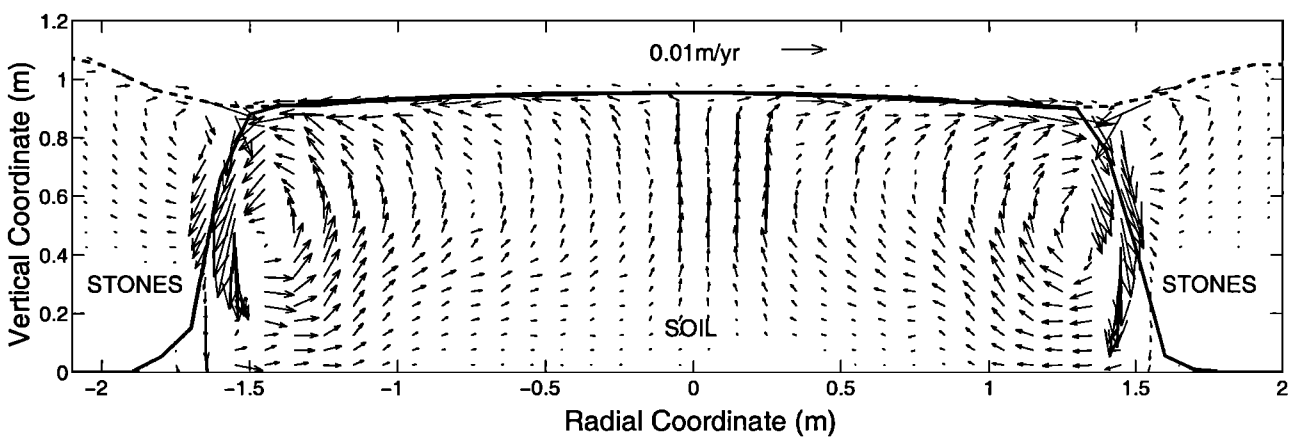

Figure 9. Velocity vectors in a cross section of a well-developed sorted circle from the reference model, as in Figure 8. Solid line is the soil surface; dashed line is the ground surface.

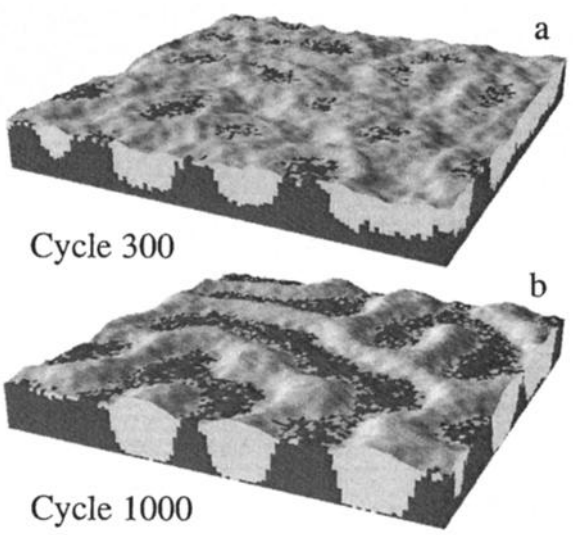

Figure 10. Sorted circles evolve to more geometrically complicated forms for volumetric fractions of soil in the active layer greater than ~0.4. Shown is a 10 m × 10 m simulation with reference model parameters, except the initial configuration is a 0.6 m thick stone layer (light shading) overlying a 0.6 m thick soil layer (dark shading), in contrast to $h_{soil} = 0.4$ m in the reference model. (a) After 300 freeze-thaw cycles circular soil plugs have risen to the surface. (b) After 1000 freeze-thaw cycles soil domains have coalesced, forming a labyrinthine pattern.

$w > \sim 0.05$ (Figure 12e). However, for $w < \sim 0.05$, negative perturbations in the stone-soil interface are unstable (increasingly unstable with decreasing w), driving soil away from soil-depleted regions, increasing the rate of soil accumulation, and decreasing the formation time (Figure 12e). Soil plug spacing and volume decrease with increasing surface temperature (closer to 0°C) during freezing because the freezing front more closely conforms to the stone-soil interface when latent heat is removed more slowly. The resulting more uniform distribution of ice particles on the stone-soil interface suppresses soil plug growth less, thereby decreasing formation time, spacing, and volume (Figure 12f).

4.2.3. Mean soil domain velocity. The mean particle velocity in the soil domain of a well-developed sorted circle is primarily dependent on the soil compressibility C (Figure 13a). Particle velocities increase with increasing C because the overall number of displacements by frost heave scales with the number of void particles and because displacement directions become less random as the density of void particles increases, thereby increasing the net circulation velocity.

Mean soil domain particle velocity decreases with time (Figure 13b) because as the stone-soil interface steepens and the diameter of the soil domain increases, inward displacements originating at the stone-soil interface, which underlie circulation, become a smaller fraction of the total number of frost-heave-induced displacements. Variations in circulation velocity with C and time overwhelm variations in velocity with other parameters.

4.2.4. Formation criteria. In the model, the parameters determining whether sorted circles form and are maintained are d_s, d_v, h_{soil}, h_{stone}, C and w. Variation in these parameters from the reference model values, one at a time, can result in several alternatives to sorted circle formation and maintenance, including a persistent unpatterned state, formation of sorted circles as a transient state in the development of other patterns, or the direct emergence of patterns other than sorted circles.

Sorted circles form if $d_s < \sim 0.8$ m. If $d_s > \sim 0.8$ m, soil accumulates into subsurface plugs that cannot penetrate the stone layer because frost-heave-induced displacements toward the surface that disperse soil overwhelm inward displacements that accumulate soil into soil plugs.

Sorted circles form if $d_v > 0.4$ m because positive perturbations in the stone-soil interface are unstable and

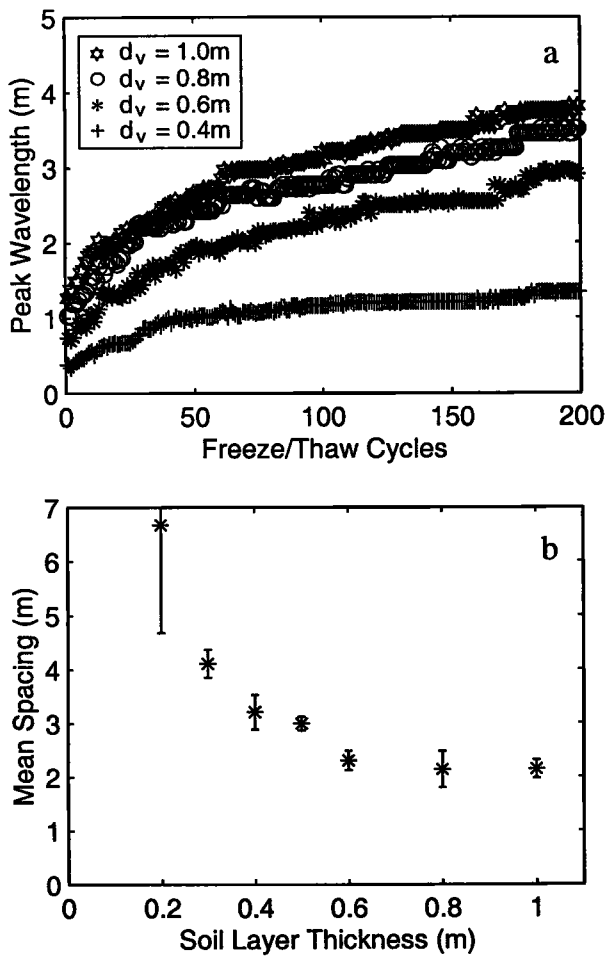

Figure 11. Dependence of spacing on d_v, time, and h_{soil}. (a) Peak wavelength of perturbations on the stone-soil interface versus time for $d_v = 0.4$-1.0 m. Increase in peak wavelength with time indicates nonlinearity. (b) Mean spacing of sorted circles versus soil layer thickness h_{soil}.

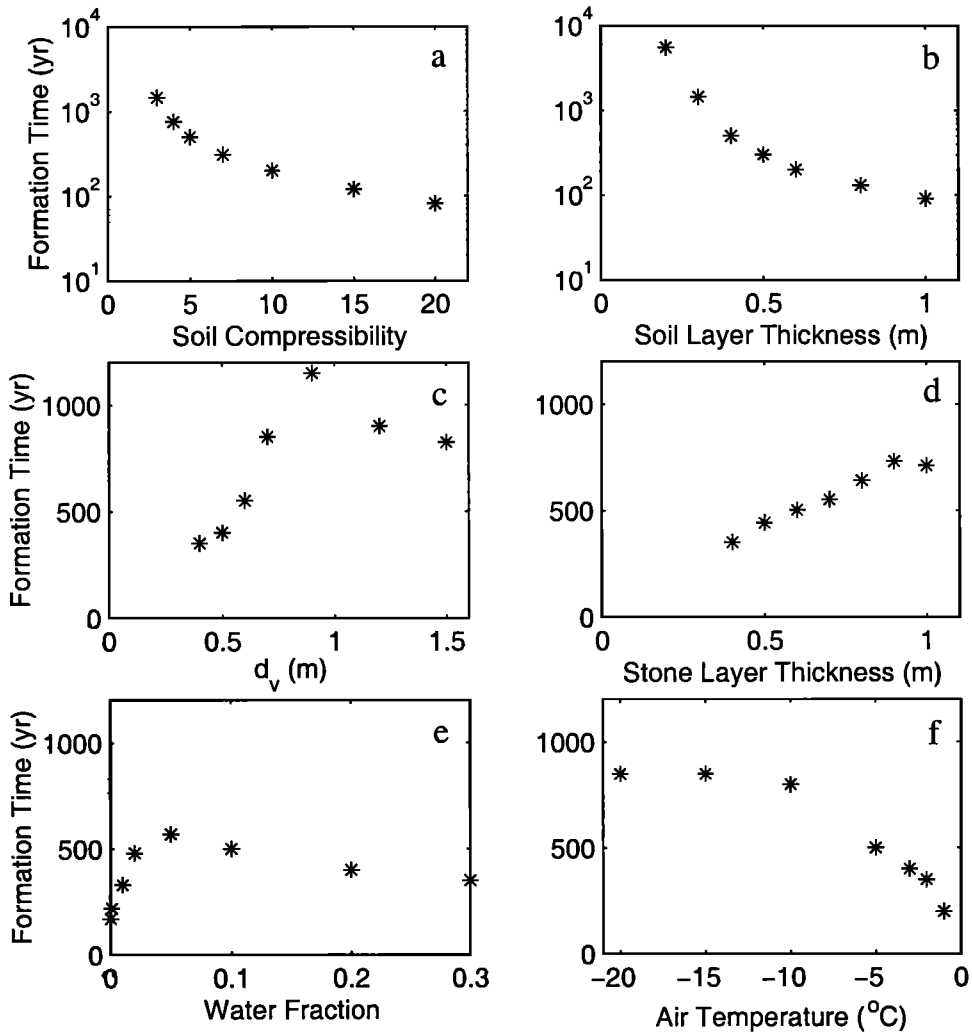

Figure 12. Dependence of formation time (number of freeze-thaw cycles until the soil domain first contacts the ground surface) on (a) soil compressibility C, (b) soil layer thickness h_{soil}, (c) maximum distance an ice particle can displace particles to a void cell d_v, (d) stone layer thickness h_{stone}, (e) volumetric fraction of water w, and (f) air temperature T_a.

develop into soil plugs. If $d_v < 0.4$ m, negative perturbations in the stone-soil interface are unstable, resulting in penetration of stone plugs to the base of the active layer rather than soil plugs to the ground surface. In this case, sorted circles do not form.

Stable sorted circles form if $\sim$0.1 m $< h_{\text{soil}} < \sim$0.6 m. If $h_{\text{soil}} < \sim$0.1 m, an insufficient supply of soil exists for soil plugs to rise to the surface. If $h_{\text{soil}} > \sim$0.6 m, sorted circles are a transient pattern, with mergers between sorted circles leading to the development of labyrinthine sorted features (Figure 10).

Sorted circles form if $h_{\text{stone}} > 0.4$ m. If $h_{\text{stone}} < 0.4$ m, small-amplitude soil plugs can form, but they do not rise to the surface because displacements toward the surface caused by frost heave, which disperse soil, overwhelm inward displacements, which accumulate soil (as for $d_s > 0.8$ m). Sorted circles form if h_{stone} is comparable to or greater than d_s ($\equiv 0.6$ m for the reference model).

Sorted circles form if $C > \sim$0.01. If $C < \sim$0.01, frost-heave-induced displacements toward the surface transporting soil away from the soil plugs dominate over soil accumulation through rare downward displacements toward sparsely distributed void cells.

Stable sorted circles form if $w > \sim$0.02. If $w < 0.02$, sorted circles repeatedly form, expand radially, and collapse. This transient behavior reflects the interplay between fast soil accumulation and processes that disperse soil after surface morphology relaxes. For small w (minimal latent heat release), the freezing front travels at nearly the same speed in the stone and soil domains; the morphology of the freezing front reflects the ground surface morphology instead of the stone-soil interface morphology (as for larger w). Initially, the freezing front morphology is coupled to the stone-soil interface morphology because perturbations in the stone-soil interface distort the ground surface. As the surface expression of the soil plug relaxes, the freezing front morphol-

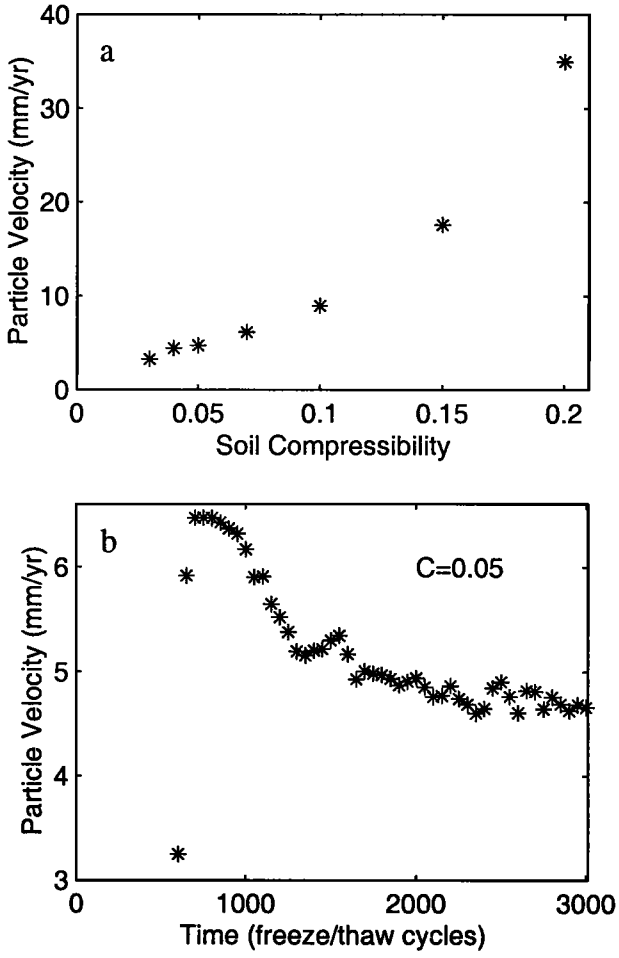

Figure 13. Dependence of mean particle velocity within soil domains on soil compressibility C and time. (a) Mean particle velocity versus C for well-developed sorted circles. (b) Mean particle velocity versus time for a developing sorted circle. Dependence of particle velocity on C and time overwhelms dependence on other parameters.

ogy decouples from the stone-soil interface morphology, and the soil plug freezes earlier and with greater frost heave than the surrounding region. As a result, the soil is displaced away from the soil plug, leading to its collapse.

4.3. Permafrost–Active Layer Interface

The emergence of features resembling sorted circles in the model, starting from an unpatterned state, indicates that the mechanisms included in this model are sufficient to produce the pattern without prior establishment of a template through additional mechanisms. To investigate the hypothesized influence of a morphological template pattern in the base of the active layer on sorted circle formation [*Ray et al.*, 1983; *Gleason et al.*, 1986; *Krantz*, 1990], two-dimensional sinusoidal perturbations, with an amplitude of 0.2 m and wavelength λ = 1-20 m, were applied to the lower boundary of the reference model. The effect of this template on sorted circle characteristics depends on the imposed wavelength relative to spacing between sorted circles in the reference model (~3.6 m). For all wavelengths, soil plugs first initiate at locations where the soil layer is thickest. If $\lambda < \sim 1.5$ m, sorted circles develop as in the reference model. If ~ 1.5 m $< \lambda < \sim 2.5$ m, sorted circles develop at the wavelength of the imposed perturbation, but then they interact and merge to form sorted circles at the reference model spacing of ~3.6 m. If ~2.5 m $< \lambda < \sim 10$ m, sorted circle spacing and location reflects the template. If $\lambda > \sim 10$ m, sorted circle spacing and location initially reflects the template, but additional sorted circles later appear between these initial forms.

Frost heave at an upward propagating freezing front has been suggested as an operative process in patterned ground [*Mackay*, 1980; *Van Vliet-Lanoe*, 1991; *Washburn*, 1997]. Temperature gradients of 2°-4°C/m have been measured at the top of the permafrost near patterned ground on Cornwallis Island [*Cook*, 1955] and beneath a sorted circle in Thule, Greenland [*Schmertmann and Taylor*, 1965]; both locations have recorded annual mean temperatures of −10° to −15°C. A series of simulations have been conducted using the processes and parameters of the reference model plus an upward propagating freezing front driven by a temperature gradient, ranging from 0.01° to 10°C/m, imposed at the base of the active layer. For temperature gradients < 4°C/m, less then 1% of the soil compression results from the upward propagating freezing front, implying that an upward propagating freezing front has negligible impact on the initiation of soil plugs or the formation of sorted circles. For large temperature gradients (> 5°C/m), the upward propagating freezing front has a significant impact on soil compression and can induce a change from sorted circle formation to stone island formation.

4.4. Upfreezing

Under repeated freezing and thawing, isolated stones migrate upward through the soil column and perpendicular to the freezing front a distance proportional to the strain induced in the surrounding soil by frost heave

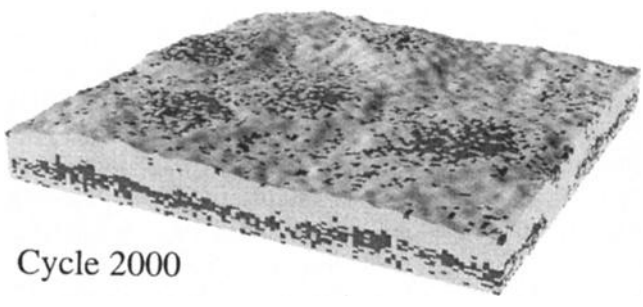

Figure 14. Reference model (10 m × 10 m) after 2000 freeze-thaw cycles with upfreezing instead of soil illuviation. Sorted circles with poorly defined stone annuli and incomplete lateral sorting develop.

[*Anderson*, 1988]. Upfreezing of stones has been hypothesized to be a contributing mechanism to the formation of sorted circles [*Washburn*, 1956, 1997], but the rate of upfreezing and the depth to which it operates, as measured in laboratory experiments, might be insufficient to produce the initial conditions of a substantial soil-free stone layer overlying a soil layer and to maintain the well-defined stone-soil interface found in sorted circles [*Prestrud*, 1987]. In the model, the initial condition of stones overlying soil and the well-defined stone-soil interface result from soil illuviation, which is assumed to operate efficiently to the base of the active layer. To test the influence of upfreezing, in lieu of illuviation, the model was modified so that (in place of the illuviation step) the contents of a cell containing a stone particle is exchanged with the contents of the cell above with a probability that is proportional to the volumetric fraction of ice in the surrounding $3 \times 3 \times 3$ cell neighborhood (equivalent to the strain induced by frost heave). All other processes and parameters were left as in the reference model, except that the initial configuration was a 1 m thick layer of randomly mixed stone and soil cells in the same ratio as the reference model. In the model, probabilities of upfreezing as low as 10% of the volumetric fraction of ice (compared to values from upfreezing experiments as high as 50% of the frost heave strain [*Anderson*, 1988]) were sufficient to produce ill-formed sorted circles. Upfreezing probabilities as high as 100% of the volumetric fraction of ice lead to patterns lacking a coherent elevated stone annulus and a well-defined stone-soil interface (Figure 14).

In this upfreezing model, sorted-circle-like patterns emerge from an initial configuration with a randomly mixed layer of stones and soil; however, sorted circles form only after substantial vertical sorting has been achieved. In all simulations presented here (including either illuviation or upfreezing), soil plugs emerge from an underlying soil layer as the precursor to sorted circles, as hypothesized in numerous discussions of sorted patterned ground [e.g., *Washburn*, 1956; *Nicholson*, 1976; *Van Vliet-Lanoe*, 1991; *Washburn*, 1997].

5. Discussion

Employing only mechanisms documented in natural active layers, forms resembling sorted circles emerge in the reference model. Sorted circles in the model and those in natural settings have many characteristics and behaviors in common, including (1) a central, circular soil plug domed upward, with diameter ~3 m, (2) a surrounding annulus of stones, ~0.5-1 m across, elevated ~0-0.5 m above the intersection with the soil domain, (3) coherent circulation in the soil domain that is upward in the center, outward at the surface, downward at the interface between stone and soil domains, and toward the center at depth, with similar circulation periods (750±350 years, ~500 years), (4) circulation within the stone domain in the opposite sense, and (5) environmental conditions consisting of annual freezing and thawing of the near-surface layer, saturated or near-saturated frost susceptible soil, and an initial mixture of stones and fine-grained soil, separated into distinct stone and soil layers.

In the model, sorted circles form by self-organization in which local and stochastic transport processes become globally coherent as the pattern develops. The formation and development of the pattern depend, in a complicated manner, on the parameters characterizing the fundamental processes of the model. However, sorted circle development is insensitive to most details of these processes because of self-organization, depending robustly on only a few critical feedbacks.

5.1. Sorted Circle Development

In the model, the same transport processes operate throughout all stages of sorted circle development. During freezing, frost heave pushes soil from the freezing front toward void cells in the unfrozen soil without regard to direction (Figure 15a). During thaw, soil compressibility is recovered by creation of void particles, resulting in vertical expansion (Figure 15b). Our interpretation of how this directional asymmetry acts to destabilize the initial stone-soil interface, form soil plugs, drive soil plugs to the surface to form sorted circles, and maintain the steady state form and behavior of sorted circles is described below.

5.1.1. Pattern initiation. In the model, sorted circles initiate through an instability in the stone-soil interface caused by frost heave. Positive perturbations in the interface grow because more soil is transported into a soil column below a perturbation by ice formation in adjacent regions than is transported to adjacent regions from ice formation within the soil column. This imbalance depends on two robust properties of the model: (1) the number of ice particles formed in a soil column (i.e., the number of transport events away from that column) is roughly uniform, irrespective of stone-soil interface morphology (Figure 15a), because the freezing front mimics this interface for small-amplitude perturbations, and (2) the number of void particles in a soil column below a perturbation (i.e., the number of transport events into that column) is greater than the number of void particles in adjacent columns (Figure 15b) because the number of void particles emplaced in a column is proportional to the soil column height (i.e., uniform distribution of void particles). This positive feedback, in which frost heave transports soil laterally toward soil-rich regions, is not reversed during thaw because consolidation of soil by removal of ice particles and expansion of soil by addition of void particles both displace soil vertically (owing to lateral confinement). Consequently, the amplitude of perturbations on the stone-soil interface increases with time.

The origin of a prominent peak in the angle-averaged, radial power spectrum of the stone-soil interface height can be explored with a linear stability analysis. For

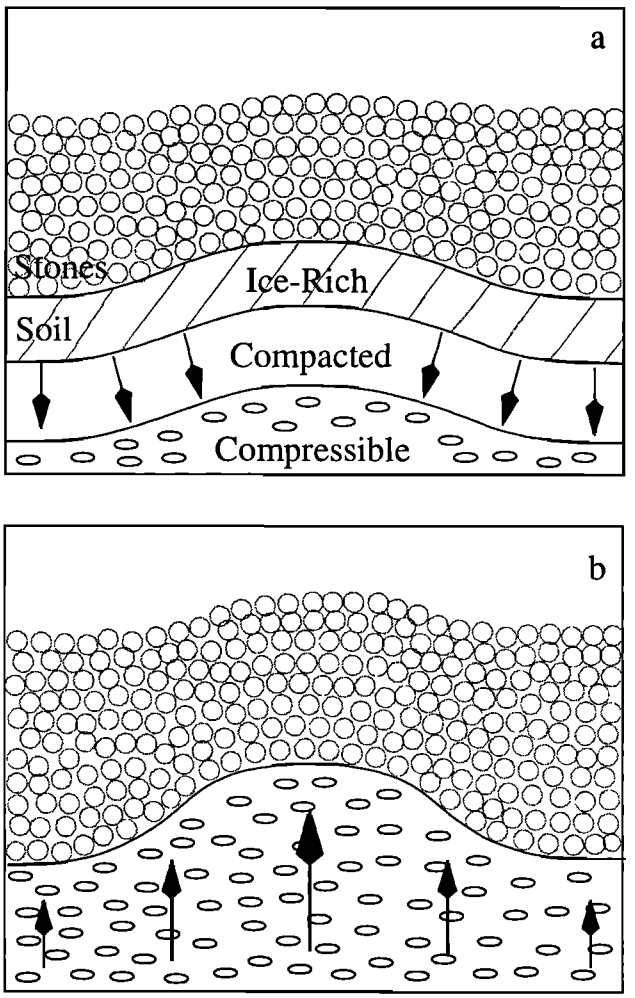

Figure 15. Instability of stone-soil interface in model (schematic cross sections). (a) Positive perturbations in the stone-soil interface. These are unstable because uniform frost heave near the stone-soil interface displaces soil laterally toward regions with taller soil columns, which have a greater number of void particles. (b) Cross section showing that during thaw, after ice-rich soil compacts, soil expands vertically by addition of void particles in proportion to the stone-soil interface height. The asymmetry between laterally biased frost-heave-induced displacements and unbiased vertical thaw compaction and soil expansion by addition of void particles results in net transport of soil toward regions where the interface height is greatest, further raising the interface.

small-amplitude perturbations the change in height of the interface at location $\boldsymbol{x_0}$ is approximately proportional to the difference between the height at $\boldsymbol{x_0}$, $\mathrm{H}(\boldsymbol{x_0})$, and a weighted average of the interface height over a distance d_v from $\boldsymbol{x_0}$. This relationship stems from mass conservation: change in height is proportional to the difference between the number of transport events bringing soil into the soil column below $\boldsymbol{x_0}$ and the number of transport events originating in and removing soil from the soil column below $\boldsymbol{x_0}$. The former is equal to the number of void particles in the column (proportional to $\mathrm{H}(\boldsymbol{x_0})$), and the latter is a weighted average of the number of void particles in the soil columns within a distance d_v, which is the average height weighted by the linearly decreasing probability of forming an ice particle with increasing distance to a void particle:

$$\frac{dH(x_0,y_0)}{dt} = C\left(H(x_0,y_0) - \frac{\int_0^{d_v}\int_0^{2\pi} H_{r\theta}\left(1-\frac{r}{d_v}\right) r d\theta dr}{\int_0^{d_v}\int_0^{2\pi}\left(1-\frac{r}{d_v}\right) r d\theta dr}\right) \quad (3a)$$

$$H_{r\theta} = H(r\cos\theta + x_0, r\sin\theta + y_0). \quad (3b)$$

Assuming a solution of the form $H = H_0 e^{\omega t + \imath(k_x x + k_y y)}$ in (3), with wave numbers k_x and k_y, yields a growth rate ω of

$$\omega = C\left(1 - \frac{6}{d_v\sqrt{k_x^2+k_y^2}}\left(J_1 d_v\sqrt{k_x^2+k_y^2} + 2\sum_{l=0}^{\infty}\frac{J_{2l+1} d_v\sqrt{k_x^2+k_y^2}}{(2l+3)(2l-1)}\right)\right), \quad (4)$$

where J_m is the Bessel function of order m. The dependence on d_v of the wavelength of the peak of this dispersion relation, approximately linear with d_v, is consistent with the dependence on d_v of the initial wavelength of the peak of the radial power spectrum of the stone-soil interface height in the model (Figure 16). However,

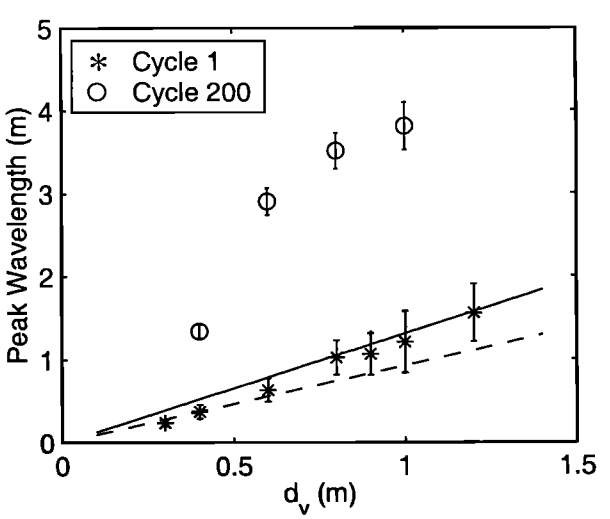

Figure 16. Peak wavelength of perturbations on the stone-soil interface after one freeze-thaw cycle and after 200 freeze-thaw cycles (when soil plugs have formed) versus d_v (peak from angle-averaged, radial power spectrum averaged over 20 independent simulations). Lines indicate the wavelength with maximum growth rate from a linear stability analysis ($\lambda = 2\pi/k_x$ from (4)). Dashed line assumes plane-wave perturbations ($k_y = 0$). Solid line assumes symmetric perturbations ($k_x = k_y$). Linear stability analysis prediction is consistent with the initial spacing, but soil plug spacing is decoupled from the initial spacing by nonlinearities.

this analysis does not address the increase in peak wavelength with time in the model (Figures 5, 11a, and 16).

5.1.2. Soil plug formation. In the model, perturbations in the stone-soil interface initiate through a positive feedback, which favors wavelengths roughly equal to d_v. The development of this instability, as described in section 5.1.1, requires that ice particles form uniformly on the stone-soil interface, a circumstance achieved only if the freezing front mimics the stone-soil interface. If the freezing front velocity in the soil domain is not negligible compared with its velocity in the stone domain, the shape of the freezing front diverges from the shape of the stone-soil interface as perturbations grow. The descending freezing front then penetrates a positive soil perturbation before reaching the stone-soil interface in surrounding regions; ice particles form first in the perturbation, pushing soil toward void cells in unfrozen soil surrounding the perturbation. This finite amplitude effect, which drives soil away from positive perturbations, is an amplitude-dependent negative feedback that favors long-wavelength features. As a result, the peak wavelength of perturbations on the stone-soil interface increases with time (Figures 5 and 16).

The increase in wavelength of perturbations on the stone-soil interface, driven by increasing negative feedback with increasing amplitude, is accomplished through mergers between initial forms. The wavelength stabilizes with further increases in amplitude because the redistribution of soil required to transition to the increasing favored wavelength cannot keep pace with the amplitude growth rate at the existing wavelength. As frost heave continues to displace soil into the core of these stabilized features, they rise toward the ground surface and develop a radially symmetric form (Figure 6) resembling soil plugs [*Washburn*, 1997]. This amplitude dependence of the length scale, indicative of nonlinearity, precludes application of a linear stability analysis to determine the spacing of either soil plugs or sorted circles.

5.1.3. Sorted circle formation and growth. As a rising soil plug approaches the ground surface, outward and upward transport by frost heave results in radial expansion of the soil domain, relaxation of the mound of stones elevated by the rising soil plug, and creation of an elevated stone annulus around the soil domain. After a soil plug contacts the ground surface, its radial expansion is accelerated by outward soil creep on the convex soil domain surface formed through frost-heave-induced displacement of soil from the flanks of the soil domain to its center. Relatively rapid radial expansion of soil domains continues until all soil from the underlying soil layer has been depleted (Figures 4 and 7).

An additional mechanism by which sorted circle soil domains expand or contract involves exchanges of soil through a network of soil conduits connecting adjacent sorted circles at the base of the active layer (Figure 17).

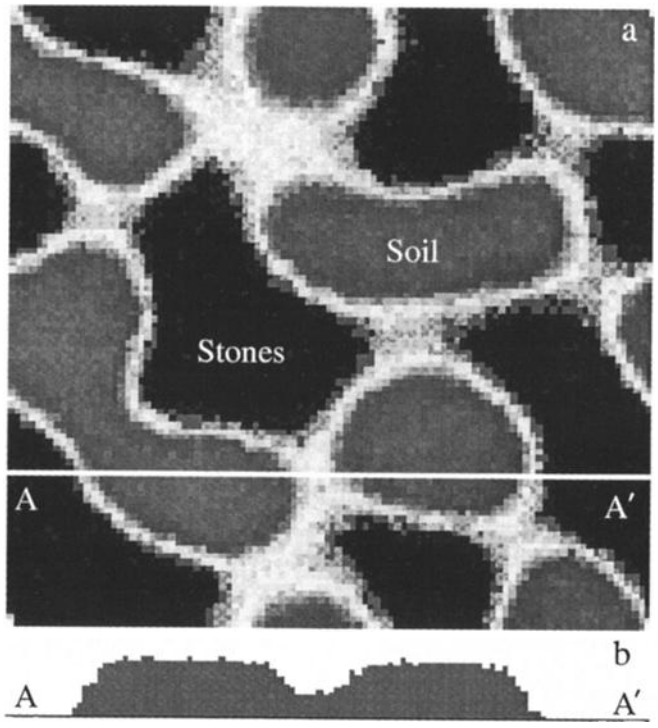

Figure 17. (a) Map of interface type for a 10 m × 10 m simulation with reference model parameters after 1300 freeze-thaw cycles: interface between soil domains and the ground surface (medium grey shading), interface between stone and soil domains (light grey or white shading), and interface between stone domains and the active layer base (solid black shading). (b) Cross-sectional view through A-A', showing soil only. Soil domains of neighboring sorted circles are connected at depth by soil conduits, which enable planform pattern changes through transfer of soil to, from, and through the conduits.

The order in which soil domains and soil conduits freeze determines which features grow at the expense of their neighbors. Those that freeze first expel soil into unfrozen adjacent features. A small sorted circle connected by a conduit to a larger sorted circle freezes more rapidly, displacing soil through the conduit to the larger soil domain and ultimately giving rise to a single, large sorted circle. Sorted circles of approximately the same size connected by a soil conduit expel soil into the conduit if they freeze first or receive soil from the conduit if the conduit freezes first. The length of the soil conduit (the distance between soil domain edges) is the primary factor determining the order of freezing. If the sorted circle soil domains freeze first (generally characterized by shorter conduits), an oblate sorted circle develops centered on the conduit. If the conduit freezes first (generally characterized by longer conduits), the conduit is severed, and the two isolated sorted circles evolve independently.

The long-term spacing between sorted circles depends, in a complicated manner, on the initial spacing of soil

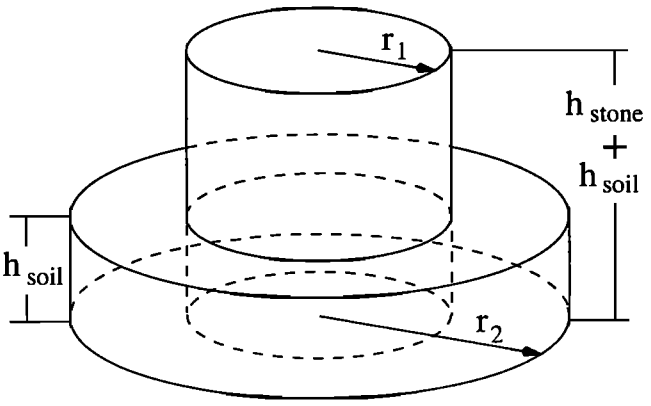

Figure 18. A simple model for soil domain radius and minimum sorted circle spacing. An inner soil cylinder, the height of the active layer, expands by depleting a disk-shaped region in the underlying soil layer. Assuming soil can be transported to the cylinder from a maximum distance of d_v, expansion continues until $r_2 - r_1 = d_v$. The final soil domain radius follows from the condition that $\pi {r_1}^2(h_{\text{soil}} + h_{\text{stone}}) = \pi {r_2}^2 h_{\text{soil}}$. The minimum center-to-center spacing of noninteracting soil domains is then $2(r_1 + d_v)$.

plugs and interactions between neighboring soil domains. However, a model that predicts the soil domain radius and the minimum distance between noninteracting sorted circles can be constructed from geometrical arguments (Figure 18). Assuming that the soil domain of a sorted circle is a cylinder of soil with radius r_1 and height equal to the active layer depth $(h_{\text{soil}} + h_{\text{stone}})$, a soil domain expands by depleting the underlying soil layer to a distance d_v from the edge of the soil domain $(r_2 = r_1 + d_v)$, and soil volume is conserved $(\pi {r_1}^2(h_{\text{soil}} + h_{\text{stone}}) = \pi {r_2}^2 h_{\text{soil}})$, the resulting soil domain radius is

$$r_1 = \frac{d_v}{\sqrt{\frac{h_{\text{soil}}+h_{\text{stone}}}{h_{\text{soil}}}} - 1}. \tag{5}$$

For reference model parameters the predicted soil domain radius is 1.03 m, which is consistent with the 1.2 ± 0.3 m mean soil domain radius in the reference model. The minimum center-to-center spacing between noninteracting soil domains $(2(r_1 + d_v))$ is 3.3 m, consistent with the 3.6 ± 0.6 m mean spacing in the reference model. This simple model is limited in that it only applies when the volumetric fraction of soil is large enough that sorted circles abut but not so large that interactions between soil plugs or sorted circles lead to labyrinthine sorted patterns. With a thinner soil layer, soil accumulates more slowly, and soil plugs remain at depth as long-wavelength, low-amplitude features. The gradation in soil column height on the flanks of a soil plug results in a flux of soil toward the soil plug over distances much greater than d_v. To predict the length scale of soil plugs and sorted circles in this case, a model for the interplay between the negative feedback driving soil away from soil plugs and the positive feedback accumulating soil will be required.

5.1.4. Maintenance and dynamics of well-developed sorted circles. In the model, sorted circles in an approximate steady state are characterized by a convex circular soil domain surrounded by an elevated stone annulus with a distinct crestline and slopes at the angle of repose. The well-defined interface between stone and soil domains is maintained by soil illuviation. The surface morphology is maintained dynamically, with both stone and soil domains circulating upward in their centers and downward at their peripheries (Figures 9 and 19).

In the soil domain, frost heave forces soil from the stone-soil interface toward the center of the soil domain

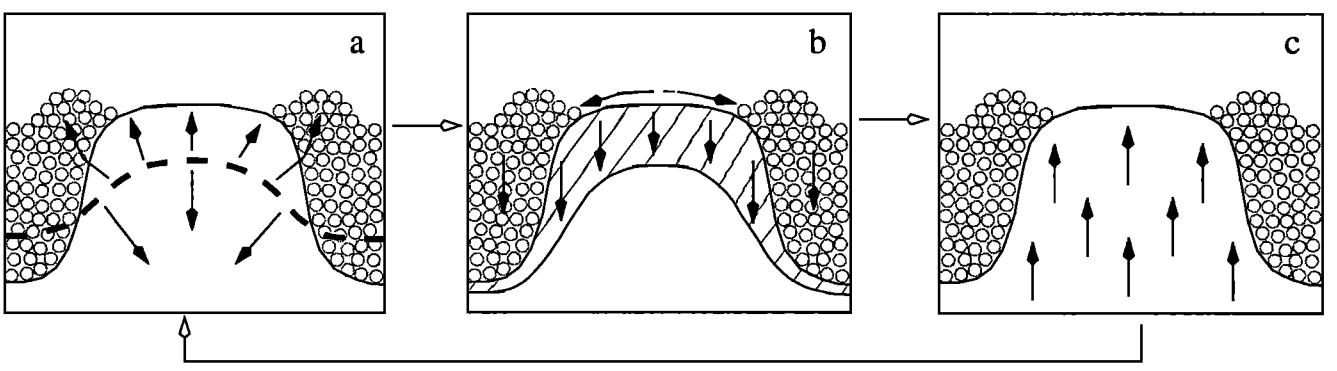

Figure 19. Maintenance of a fully developed sorted circle in the model (schematic cross sections). Solid arrows indicate transport direction during each stage of the freeze-thaw cycle. (a) Cross section showing that during freezing, frost heave displaces material away from the freezing front (dashed line): outward toward the ground surface and inward toward unfrozen soil. (b) Cross section showing that during thaw, ice-rich soil near the surface and stone-soil interface expels water and compacts vertically. Soil sifts downward through the pore space between stones (illuviation); soil on the surface creeps downslope. (c) During thaw, desiccated and compacted soil expands vertically by the addition of void particles.

(Figure 19a), expansion by addition of void particles during thaw displaces soil upward (Figure 19c), soil relaxes outward on the convex soil domain surface by soil creep (Figure 19b), and soil on the flanks of the soil domain relaxes downward by removal of ice near the stone-soil interface and by soil illuviation in the stone domain (Figure 19b), completing the loop. Frost heave originating near the stone-soil interface in the upper part of the soil domain pushes outward and upward, elevating and steepening the surface of the stone annulus primarily inside its crestline and leading to inward transport of stones on slopes in excess of the angle of repose. This results in a pattern of circulation in the stone annulus opposite in sense to the circulation in the soil domain. Rare displacements by frost heave (lower in the soil domain) to the outer slope of the stone annulus displace its crestline outward.

5.2. Further Research

A number of problems and questions requiring further research are suggested by the results of the model and the interpretations we have presented, including how the abstracted rules and parameters in the model correspond to physical processes and measurable quantities in the natural system. For example, water is not treated explicitly in the model. The volumetric fraction of water in a soil cell w determines the velocity of the freezing front in soil; however, the soil compressibility C determines the number of ice particles that form (frost heave). The two parameters are not coupled, leading to the possibility of unrealistic behavior, such as frost heave and consequent pattern formation when $w = 0$. In natural active layers, frost heave might be limited by water availability, soil compressibility, or some combination of the two. Because of the prominent role frost heave plays in sorted circle formation a more complete treatment of water and soil compressibility would significantly improve the delineation of the physical parameters controlling the occurrence and properties of sorted circles. Similarly, the physical process underlying soil expansion during thaw could be treated more accurately with regard to the spatial distribution and the numerical value of C if the relative roles of water absorption and frost heave could be ascertained. Finally, vertical sorting between stone and soil domains is a critical process in sorted circle formation. In the model, illuviation transports soil to the base of the stone layer within one freeze-thaw cycle. Determining the contributions of soil illuviation and upfreezing to sorting and testing the assumption of rapid soil illuviation, through further modeling and measurements, could help to constrain the predicted conditions under which and the speed with which sorted circles form.

In the model, sorted circle formation, size, and spacing are strongly dependent on the parameter d_v. However, this parameter does not correspond clearly with an identified property of soils, nor has the functional dependence of ice formation on d_v been modeled or measured. In the model, soil displacements in sorted circles are coherent over a distance d_v, which is typically 25-50% of the soil domain diameter. Further progress on modeling sorted circles will require determining how ice lenses forming within a freezing front coherently displace soil over this range, perhaps using a model for stresses and strains within a partially frozen, partially saturated heterogeneous soil.

Because displacement paths of neighboring ice particles are determined sequentially and individually, the model introduces small-scale randomness and mixing that are not expected to be present in natural active layers, where frost-heave-induced displacements should be more coherent. Consequently, comparison between particle velocities in the model and measurements from natural sorted circles has required averaging over many simulations with different sequences of random numbers. Further and better comparisons might be facilitated by a model in which correlation of displacements is enforced.

In addition to exploring the range of conditions under which sorted circles form and are stable, variation of model parameters has indicated that a broader range of sorted patterns (i.e, labyrinthine sorted features, stone islands, and downward stone plugs) might be formed by these processes under different conditions, such as high volumetric fractions of soil, small d_v, low volumetric water fractions w, or an upward propagating freezing front. Moreover, a range of sorted patterns found in natural active layers do not occur in the model, including sorted polygons. Progress and insights into which mechanisms and parameters underlie the transition between patterns could be gained from further exploration of this model and by developing models with additional or differing active layer processes.

6. Conclusions

Numerous hypotheses for the initiation, development, and long-term maintenance of sorted circles have been proposed; however, most of these hypotheses have not been modeled in sufficient detail that differing mechanisms can be compared. We have presented a numerical model for all stages of sorted circle development, in which freezing front propagation, frost heave, soil illuviation, surface creep, compaction during thawing, and soil expansion by water absorption act to transport stones and soil in a simulated active layer during an annual freeze-thaw cycle.

Many of the qualitative and quantitative characteristics of sorted circles are reproduced: (1) ~3 m wide convex circular soil domains, which emerge from a laterally uniform active layer after several hundred freeze-thaw cycles, (2) an encircling 0.5-1.0 m raised annulus of stones, and (3) coherent circulation in both stone and soil domains, with period ~750 years in the soil domain.

A robust positive feedback operates throughout all stages of sorted circle development. Expansion by frost heave at the interface between stone and soil domains transports soil toward the soil domain and stones toward the stone domain. Acted upon by this mechanism, a uniform layer of stones devoid of soil overlying a uniform layer of soil is unstable, leading to the development of soil plugs. A negative feedback, related to the decoupling of the freezing front from the stone-soil interface as finite amplitude features emerge, causes soil plugs to grow radially and merge, while the positive feedback drives them to the surface, pushes up stone rings around the emerging soil plugs, and maintains long-term circulation within the stone and soil domains. The finite amplitude, nonlinear effects of the negative feedback render a linear stability analysis of the patterns inapplicable. The robust feedbacks that give rise to the self-organized pattern of sorted circles have been treated here. The formation of other types of sorted patterned ground will be explored in future work.

Acknowledgments. We are grateful to Lawrence Plug, Glenn Ierley, Linden Clarke, Michele Okihiro, and Giovanni Coco for many enlightening conversations, mathematical insights and helpful advice. This work was supported by the National Science Foundation, Arctic Natural Sciences Program (OPP-9530860 to B.T.W. and OPP-9530809 to B.H.) a National Defense Science and Engineering Graduate Fellowship (to M.A.K.) and the Andrew W. Mellon Foundation (to B.T.W.).

References

Anderson, S. P., The upfreezing process: Experiments with a single clast, *Geol. Soc. Am. Bull.*, *100*, 609–621, 1988.

Combarnous, M. A., and S. A. Bories, Hydrothermal convection in saturated porous media, *Adv. Hydrosci.*, *10*, 231–307, 1975.

Cook, F. A., Near surface soil temperature measurements at Resolute Bay, Nortwest Territories, *Arctic*, *8*(1), 237–249, 1955.

Corte, A. E., The frost behavior of soils: Laboratory and field data for a new concept, part I, Vertical sorting, technical report, U. S. Army Cold Regions and Eng. Lab., Hanover, N. H., 1961.

Corte, A. E., Particle sorting by repeated freezing and thawing, *Biul. Peryglacjalny*, *15*, 175–240, 1966.

DuFort, E. C., and S. P. Frankel, Stability conditions in the numerical treatment of parabolic differential equations, *Math. Tables Other Aids Comput.*, *7*, 135-152, 1953.

Elton, C., The nature and origin of soil polygons in Spitsbergen, *Geol. Soc. London Q. J.*, *83*, 163–194, 1927.

Forman, S. L., and G. H. Miller, Time-dependent soil morphologies and pedogenic processes, *Arctic Alpine Res.*, *16*(4), 381–394, 1984.

Gleason, K. J., W. B. Krantz, N. Caine, J. H. George, and R. D. Gunn, Geometrical aspects of sorted patterned ground in recurrently frozen soil, *Science*, *232*, 216–220, 1986.

Goldthwait, R. P., Frost sorted patterned ground: A review, *Q. Res.*, *6*, 27–35, 1976.

Hallet, B., Measurement of soil motion in sorted circles, western Spitsbergen, in *Proceedings of the Seventh International Conference on Permafrost*, edited by A. G. Lewkowicz, 57, pp. 415–420, Collect. Nordicana, 1998.

Hallet, B., and S. Prestrud, Dynamics of periglacial sorted circles in western Spitsbergen, *Q. Res.*, *26*, 81–99, 1986.

Hallet, B., and E. D. Waddington, Buoyancy forces induced by freeze-thaw in the active layer: Implications for diapirism and soil circulation, in *Periglacial Geomorphology*, edited by J. C. Dixon and A. D. Abrahams, pp. 251–279, John Wiley, New York, 1991.

Hallet, B., S. P. Anderson, C. W. Stubbs, and E. C. Gregory, Surface soil displacements in sorted circles, western Spitsbergen, in *Proceedings of the Fifth International Conference on Permafrost*, vol. 1, edited by K. Senneset, pp. 770–775, Tapir, Trondheim, Norway, 1988.

Jyotsna, R., and P. K. Haff, Microtopography as an indicator of modern hillslope diffusivity in arid terrain, *Geology*, *25*(8), 695–698, 1997.

Krantz, W. B., Self-organization as patterned ground in recurrently frozen soils, *Earth Sci. Rev.*, *29*, 117–130, 1990.

Lambe, T. W., and R. V. Whitman, *Soil Mechanics*, John Wiley, New York, 1969.

Mackay, J. R., An equilibrium model for hummocks (non-sorted circles), Garry Island, Northwest Territories, *Pap. Geol. Surv. Can.*, *79-1A*, 165–167, 1979.

Mackay, J. R., The origin of hummocks, Western Arctic coast, Canada, *Can. J. Earth Sci.*, *17*, 996–1006, 1980.

Mortensen, H., Uber die physikalische moglichkeit der "brodel" -hypothese, *Centralblatt fur Mineral. Geol. Palaontol., Abt. B*, *1932*, 417–422, 1932.

Nicholson, F. H., Patterned ground formation and description as suggested by low Arctic and sub-Arctic examples, *Arctic Alpine Res.*, *8*, 329–34, 1976.

Nicolis, G., and I. Prigogine, *Self-Organization in Nonequilibrium Systems: From Dissipative Structures to Order Through Fluctuations*, John Wiley, New York, 1977.

Nogotov, E. F., *Applications of Numerical Heat Transfer*, U. N. Educ., Sci., and Cult. Org., Paris, France, 1978.

Pissart, A., Advances in periglacial geomorphology, *Z. Geomorphol.*, *79*, 119–131, 1990.

Prestrud, S., The upfreezing process and its role in sorted circles, M.S. thesis, Univ. of Wash., Seattle, 1987.

Putkonen, J., Soil thermal properties and heat transfer processes near Ny-Alesund, Northwestern Spitsbergen, Svalbard, *Polar Res.*, *17*(2), 165–179, 1998.

Ray, R. J., W. B. Krantz, T. N. Caine, and R. D. Gunn, A model for sorted patterned-ground regularity, *J. Glaciol.*, *29*(102), 317–337, 1983.

Schmertmann, J., and R. S. Taylor, Quantitative data from a patterned ground site over permafrost, technical report, U.S. Army Cold Regions and Eng. Lab., Hanover, N.H., 1965.

Taber, S., Frost heaving, *J. Geol.*, *37*, 428–461, 1929.

Van Vliet-Lanoe, B., Differential frost heave, loadcasting and convection, converging mechanisms, a discussion of the origin of cryoturbations, *Permafrost Periglacial Processes*, *2*, 123–139, 1991.

Washburn, A. L., Classification of patterned ground and review of suggested origins, *Geol. Soc. Am. Bull.*, *67*, 823–865, 1956.

Washburn, A. L., Weathering, frost action, and patterned ground in the Mesters Vig District, northeast Greenland, *Medd. Groenl.*, *176*, 259–330, 1969.

Washburn, A. L., *Geocryology: A Survey of Periglacial Processes and Environments*, Halsted, New York, 1980.

Washburn, A. L., *Plugs and Plug Circles: A Basic Form of Patterned Ground, Cornwallis Island, Arctic Canada-Origin and Implications*, *Mem. Geol. Soc. Am.*, 190, 1997.

Werner, B. T., Eolian dunes: Computer simulations and attractor interpretation, *Geology*, *23*, 1107–1110, 1995.

Werner, B. T., Complexity in natural landform patterns, *Science*, *284*, 102–104, 1999.

Williams, P. J., and M. W. Smith, *The Frozen Earth: Fundamentals of Geocryology*, Cambridge Univ. Press, New York, 1989.

M. A. Kessler and B. T. Werner, Complex Systems Laboratory, Cecil and Ida Green Institute of Geophysics and Planetary Physics, University of California-San Diego, La Jolla, CA 92093-0225. (kessler@ucsd.edu)

A. B. Murray, Division of Earth and Ocean Science/Center for Nonlinear and Complex Systems, Duke University, Durham, NC 27708-0230.

B. Hallet, Quaternary Research Center and Department of Geological Sciences, University of Washington, Seattle, WA 98195-1310.

(Received August 28, 2000; revised January 31, 2001; accepted March 25, 2001.)