SIO223A: Geophysical Data Analysis
Course Outline

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1 Linear algebra

Lecture 1: (Basics) *Vectors*: rows and columns, linear (in)dependence, norms, span, orthogonal vectors. *Matrices*: multiplication/addition of matrices, rank, matrix norms and induced norms, (equivalence of norms in finite dimensional spaces), null-space, column-space, row-space. *Special matrices*: square (invertible), symmetric, symmetric positive definite, skew-symmetric, orthogonal.

Lecture 2: Linear systems ($Ax = b$): Row-reduction (Gaussian elimination), LU-factorization, Gauss-Seidel as an example of an iterative method.

Lecture 3: Least squares ($\min_x ||Ax - b||$): Normal equations, Gram-Schmidt (QR), examples of least squares problems.

Lecture 4: Eigenvalues & eigenvectors ($Av = \lambda v$): Definitions, spectral decomposition, diagonalization, low-rank approximations, power method. Singular value decomposition (SVD) and its use in data compression (low-resolution images). SVD as a tool for solving least squares problems.

References: Lancaster (1966), Horn & Johnson (1985), Strang (2006).

2 Stochastic methods

Lecture 5: Definitions of probability, random variables, distribution and density functions.

Lecture 6: Examples of random variables: normal, uniform, exponential, Poisson (discrete), gamma, χ^2, student-t, log-normal.

Lecture 7: Expected values and moments (mean, standard deviation, skewness, kurtosis). The expected value is an optimal estimate.

Lecture 8 & 9: Multi-variate distributions (general), joint distributions, marginals and independence. Conditional probability and Bayes' rule. Linear transformations and change of variables. Sums of random variables and the central limit theorem.

Lecture 10 & 11: Examples of multivariate distributions: normal, uniform (cube), uniform (sphere). For normal distributions: covariance, affine transformations, removing correlations (and PCA), conditionals (conditional mean and variance/regression).

Lectures 12 & 13: Estimating means and variances, desirable properties of estimators (unbiased, consistent), the boot-strap, how to deal with outliers, maximum likelihood estimators.

Lecture 14: Introduction to stochastic processes: Definition and stationarity. Examples: (i) Brownian motion (mathematics version); (ii) Gaussian processes. More of this in 224B.

Lecture 15 & 16: Direct sampling/importance sampling, Markov chain Monte Carlo.

References: Chapters 1-5 of class notes; Chorin & Hald (2013); Rasmussen & Williams (2006); Owen (2013). For sampling and MCMC: MacKay (1998); Owen (2013) and Jonathan Goodman's lecture notes.

3 Curve fitting and interpolation

Lecture 17: Least squares revisited - analyzing error statistics.

Lecture 18: Splines: interpolation and fitting linear splines.

Lecture 19: Gaussian process regression. Basics of GP regression, conditionals, sampling. Hyper parameters and sensitivity to hyper parameters. Estimation of hyper parameters.

Time permitting: Nonlinear least squares (Gauss-Newton). Total least squares.

References: Class-notes, Rasmussen & Williams (2006), Lancaster & Šalkausas (1986) and Nocedal & Wright (2006).

Homework

Homework is assigned every week and consists of exercises that are done with pen and paper and each homework will also include a programming task. Homework solutions are discussed in class.

References

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MacKay, D., 1998. Introduction to Monte carlo methods, *Learning in Graphical Models, NATO Science Series*, **89**, 175–204.

Nocedal, J. & Wright, S., 2006. *Numerical Optimization*, Springer, 2nd edn.

Owen, A., 2013. *Monte Carlo theory, methods and examples*.

Rasmussen, C. & Williams, C., 2006. *Gaussian Processes for Machine Learning*, The MIT Press.

Strang, G., 2006. *Linear Algebra and its Applications*, Brooks/Cole.