SIOG 229: Gravity and Geomagnetism

Winter 2021, Instructors A. Borsa, C.Constable, Meets MWF 10:15-11:15.

Overview

SIO 229 is an introduction to potential theory in the context of gravity and geomagnetism at a level suitable for the advanced nonspecialist in geophysics. The subtext is to get you familiar with spherical harmonics, which are ubiquitous functions in global geophysics. After a brief warm-up we are going to introduce spherical harmonics, derive some of their properties and apply them to various problems in potential fields, with emphasis on the geoid. Along the way we will meet some old topics, like the precession of the equinoxes and classical problems of potential theory, as well as some newer ones, like satellite-derived geoid and gravity models of other bodies in the solar system.

Theoretical topics include multipole expansions, spherical harmonics, Laplace's equation, boundary-value problems on a sphere, and Fourier methods. Applied topics include the global geoid, gravity anomalies, geomagnetic field modeling and sources, and paleomagnetic observations.

Reading

Required course reading will be in the form of several sets of notes (authored by Adrian Borsa, Cathy Constable, and Bob Parker) with assigned sections for each lecture posted in Canvas.

Grading

Grading for this class will be based primarily on homework assignments. Students may work together on assignments, but each student must turn in their own solutions based on their own work. Any code written to complete assignments (e.g. Matlab routines) should be submitted along with results. Short presentations of topics from the text or homework may be assigned, and a final exam may be given at the instructors' discretion.

Supplementary References

Physical Geodesy
W. Heiskanen and H. Moritz
W.H. Freeman and Company, San Francisco, 1967
A thorough treatment of much of the gravity material covered in the class notes.

Treatise on Geophysics, Volume 3, Geodesy
Volume Editor: T. Herring. Editor-in-Chief: G. Schubert
Elsevier, Amsterdam, 2007
Technical summaries of many current topics in gravity research, written by the best in the field.

Potential Theory in Gravity and Magnetic Applications
Richard J. Blakely
Cambridge University Press, New York 1995
Another look at potential theory, specifically in the context of gravity and geomagnetism.

Foundations of Geomagnetism, George Backus, Robert Parker, Catherine Constable, Cambridge University Press, New York 1996

Introduction to Electrodynamics, David J. Griffiths, 4th Edition, Cambridge University Press, 2017. Undergraduate physics text – good for a refresher on basic electromagnetism

Treatise on Geophysics, Volume 5, Geomagnetism, Volume Editor: M. Kono. Editor-in-Chief: G. Schubert, Elsevier, Amsterdam, 2015, 2nd Edition. [available at https://www.sciencedirect.com/referencework/9780444538031/treatise-on-geophysics in electronic format] Provides technical summaries of many current topics in geomagnetic research.

Treatise on Geophysics, Volume 8, Core Dynamics, Volume Editor: P. Olson. Editor-in-Chief: G. Schubert, Elsevier, Amsterdam, 2015, 2nd Edition. [available at https://www.sciencedirect.com/referencework/9780444538031/treatise-on-geophysics in electronic format]

Space Science reviews volume on *Planetary Magnetism* at
http://link.springer.com/journal/11214/152/1/page/1 (Links to an external site.)

(Links to an external site.)and on *Terrestrial Magnetism*
http://link.springer.com/journal/11214/155/1/page/1 (Links to an external site.)

Earth's Electromagnetic Environment Surv Geophys DOI 10.1007/s10712-015-9351-1 (Links to an external site.)

Part 1 Gravity

Lecture 1: (1/4/2021)
Introduction to the gravity portion of the course; a brief look at gravity via the Earth and Moon.

Lecture 2: (1/6/2021)
Formal definitions for gravity and gravitational potential. Coordinate systems and the del (gradient) operator.

Lecture 3: (1/8/2021)
Introduction to moments of inertia. Moment of inertia for an arbitrary body.

Lecture 4: (1/11/2021)
Principal axes of the inertia tensor. Angular momentum of Earth and the Chandler Wobble.

Lecture 5: (1/13/2021)
Multipole expansion of the gravitational potential of an arbitrary body.
Homework (due Friday, January 22):

Lecture 6: (1/15/2021)
MacCullagh's Formula and the moments of inertia of Earth.

Lecture 7: (1/20/2021)
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Lecture 8: (1/22/2021)
A first look at the geoid.

Lecture 9: (1/25/2021)
A simple ellipsoidal geoid: Clairaut's Formula.

Lecture 10: (1/27/2021)
Spherical harmonics as a general solution to Laplace's equation.

Lecture 11 (1/29/2021)
A Field Guide to the Spherical Harmonics.

Lecture 12 (2/1/2021)
Spherical Harmonics adapted to Earth's gravitational potential field. Table of Spherical Harmonic Lore.

Lecture 13 (2/3/2021)
Spherical harmonic models of Earth's gravity field. A more realistic geoid.

Lecture 14 (2/5/2021)
Gravity of Earth and beyond.

Part 2: Geomagnetism

Lecture G1 (2/8/2021) - Introduction to Planetary & Geomagnetism

Lecture G2 (2/10/2021) - Classical Electrodynamics

Geomagnetism Problem Set 1

Lecture G3: (2/12/2021) Constitutive relations and neglecting the displacement current. When can we write the magnetic field as the gradient of a scalar potential and use Laplace's equation?

Continue reading pages 1-14 of the notes, and make sure you refresh your skills in electrodynamics. See Chapter 2 of Backus et al., Foundations of Geomagnetism for an expanded

version of lectures G2 and G3, or the book by Griffiths on Introduction to Electrodynamics. Wikipedia articles on electric and magnetic fields also provide a crash course for those who like their readings to be short.

Lecture G4: (2/17/2021) Gauss' Theory of the main field, separation into internal and external parts, upward and downward continuation.

Lecture G5:(2/19/2021) Constructing Geomagnetic Field Models.

Read the notes on Gauss' theory of the main field and constructing geomagnetic field models.

Geomagnetism Problem Set 2

Lecture G6 (2/22/21): We will be continuing our discussion of how to make regularized field models with different norms, look at some results, and introduce the spatial geomagnetic spectrum.

Lecture G7 (2/24/2021) Today's class is on secular variations and will also introduce poloidal and toroidal fields highlighting the limitations of Gauss' theory for the core.

Lecture G8 (2/26/2021) Ingredients for a geodynamo -Magnetohydrodynamics in the Core

Lecture G9 (3/1/2021)Magnetohydrodynamics in the Core- part II

Read class notes on magnetohydrodynamics

Lecture G10 (3/3/2021) The lithospheric magnetic field.

Lecture G11 (3/5/2021) The lithospheric magnetic field continued - the magnetic annihilator and Runcorn's Theorem. For much more on crustal magnetism consult Chapter 6 of Volume 5 of the Treatise on Geophysics.

Geomagnetism Problem Set 3

Lecture G12 (3/8/2021): Electromagnetic induction in Earth's crust and mantle. Background reading in Chapter 7 of Vol 5 of the Treatise on Geophysics.

Lecture G13, G14 (3/10 and 3/12/2021) Class presentations on magnetic topics