SIOG 230 Introduction to Inverse Theory
Syllabus

Instructor: Cathy Constable (cconstable@ucsd.edu, x43183)

4 units, 3 hour meeting/week, homework, S/U grades permitted.

Prerequisites: graduate standing or consent of instructor; familiarity with topics covered in SIOG 223A.

Spring 2021, Tuesday and Thursday afternoons 2:00-3:20 pm.

Description: This class deals with geophysical inverse theory, but is often also of interest to others interested in making inferences about the Earth or other systems underlying observations that are both finite in numbers and accuracy. The key factor that makes inverse theory different from simple parameter estimation (the classical statistical problem) is that while the number of observations available is finite, the unknown model requires infinitely many variables for its full description. Thus in practical inverse problems (those based on real as opposed to idealized data) there is always ambiguity in the model. Finding a particular model solution involves a choice from an infinitely large collection of alternative solutions. The initial approach taken in the class is to apply mathematical optimization to select the simplest models. This avoids the introduction of unnecessary exciting features. Uncertainty quanitification is also an important goal. To discover reliable properties of the earth, independent of any particular model, we may need to calculate upper and lower bounds on functionals that represent those properties. The utility of stochastic methods for identifying a range of acceptable models will also be discussed.

The class will begin with a broad overview of least squares estimation, inversion, and some practical applications, followed by a synopsis of some necessary mathematical ideas. There will be homework, which will be posted on the class website: http://igppweb.ucsd.edu/∼cathy/Classes/SIO230/index.html

Other information will also be posted here. The course draws on several classic texts on inverse theory, and class notes supplied as we go along. Further books for background reading on mathematical and statistical matters are listed below. Early in the course students are expected bring to class an example of a forward problem of interest to them. Be prepared to explain it to the rest of the class. Later work will involve evaluating how class material might help with solving the inverse problem.

Topics included:

(1) Mathematical Precursors, Linear algebra and Vector spaces

(2) Over and Underdetermined least squares problems

(3) Linear inverse problems with exact and uncertain data; model construction; regularization as the minimization of model complexity.

(4) Numerical methods for practical solutions, including QR and SVD factorizations.

(5) Resolution & Inference: averaging or bounds on models

(6) Other constraints; linear and quadratic programming.

(7) Nonlinear inverse problems solved by linearization; Fréchet and Gateaux derivatives.

(8) Iterative optimization: Backus-Gilbert creeping; Occam's method.

(9) Stochastic inversion, trans-dimensional inversion, and other topics

Recommended Reading

Aster, R.C., B. Borchers, & C.H. Thurber, 2013. *Parameter Estimation and Inverse Problems, Second Edition*. Academic Press, Oxford, UK.

Parker, R.L., 1994. *Geophysical Inverse Theory*. Princeton University Press, Princeton, New Jersey.

Tarantola, A., 2005. *Inverse Problem Theory and Methods for Model Parameter Estimation*. SIAM, Philadelphia, Pensylvania.

Strang, G., 1980. *Linear Algebra and its Applications*. Academic Press, New York.

Strang, G., 1986. *Introduction to Applied Mathematics*. Wellesley-Cambridge Press, Wellesley, Mass..

Dekking, F.M., C. Kraaikamp, H.P. Lopuhaä, L.E. Meester, 2005. *A Modern Introduction to Probability and Statistics: Understanding Why and How*. Springer, London.

Jaynes, E.T., 2003. *Probability Theory: The Logic of Science*. Cambridge University Press, New York.