Syllabus and description of SIO239 (4 units)

Computational tools for inverse problems and data assimilation.

Short description

This course covers computational methods for the solution of geophysical inverse problems and time-dependent data assimilation problems. Topics to be covered include numerical optimization, Markov chain Monte Carlo, sequential importance sampling, Kalman and ensemble Kalman filtering, particle filters and cycling variational methods. *All* numerical methods we cover find application in Earth Science, but this course focuses on the numerical and computational methods, not the physics.

The course will require frequent homework, that consists of derivations and coding assignments. We will also have a final project that goes beyond the homework. Throughout the course, all students will implement the algorithms we cover on simplified but meaningful test problems, relevant to the GP and/or CASPO curricula.

Pre-requisites. SIOG230 (Inverse theory), and/or a good foundational understanding of (i) linear algebra; (ii) basic data fitting; and (iii) random variables; or consent of instructor. Some coding experience is also necessary.

Detailed breakdown of topics

1. Rapid review of random variables, conditional probability and basic ideas of Monte Carlo.
2. Review of optimization and linear algebra.
3. Deterministic inverse problems and 4D-Var. Linear/linearized problems and the Kalman gain. Nonlinear problems and nonlinear least squares.
4. Probabilistic interpretations of optimization and extension to randomize-then-optimize.
5. Markov chain Monte Carlo (MCMC) basics: Metropolis-Hastings and random walk Metropolis.
6. MCMC diagnostics: Integrated auto correlation time and effective sample size.
7. Beyond random walk Metropolis: MALA, ensemble samplers, and function space MCMC.
8. Gibbs sampling and hierarchical Bayesian inverse problems.
9. Time dependent problems: data assimilation and the Kalman filter.
10. The ensemble Kalman filter (EnKF).
11. EnKF in practice: Localization, inflation, tuning. EnKF and BLUE in nonlinear problems.
12. Cycling 4D-Var, EDA and E4DVar.
13. Importance sampling, sequential importance sampling and particle filters.
14. How to construct "good" proposal distributions. "Standard" and optimal particle filters.
15. Limitations of sampling in high dimensions: Filter collapse and the curse of dimensionality.
16. Connections and similarities with machine learning. Stochastic gradient descent, ensemble-based optimization, neural networks.

This should make about 16 lectures (assuming a Tu/Th schedule), leaving some room for homework discussion and presentation of final projects.